MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The election by the Council of the following Fellows and Associates was duly confirmed :—

FELLOWS.

Arthur Edward Herbert Bleksley, M.Sc., Quarrywoods, Stellenbosch, S. Africa ;

Don Abraham Devendra, B.A., Trinity College, Kandy, Ceylon ;

Charles Albert Griffiths, 165 Kingsley Road, Princes Park, Liverpool ;

Herbert Nicholas Jones, 25 Commercial Road, Camberwell, London, S.E. 15 ;

Clement Frederick Press, 191 Tufnell Park Road, London, N. 7 ;

Alexander Hamilton Rice, M.A., M.D., 901 Fifth Avenue, New York City, and Miramar, Newport, R.I., U.S.A. ; and

Rev. William Lisle Shepherd, M.A., St. Mary's Vicarage, Halifax, Yorks.

ASSOCIATES.

Professor A. Pannekoek, The University, Amsterdam, Holland ;

M. Paul Stroobant, Observatoire Royal de Belgique, Uccle, Belgium.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Mark Herbert Joseph Bennett, Lieutenant, R.N.R., Hoglands Wood, Dunmow, Essex (proposed by C. E. Adams) ;

John Hasselkus, Director of Ross Limited, Optical Works, 3 North Side, Clapham, London, S.W. 4 (proposed by Charles A. Parsons) ;
Hector Archibald McDonald, B.Sc., Instructor Lieutenant Commander, R.N., Royal Naval College, Greenwich, London, S.E. 10 (proposed by L. M. Milne-Thomson) ; and
Alexander Raven Thomson, 30 Overstrand Mansions, London, S.W. 11 (proposed by W. A. Parr).

Eighty-five presents were announced as having been received since the last meeting, including—

Córdoba Observatory, Resultados, vol. xxxv. (Fundamental Catalogue).
International Astronomical Union, " Délimitation scientifique des constellations," par E. Delporte. Being the Report of Commission III.
Mount Wilson Observatory, Papers, vol. iv. (Mt. Wilson Catalogue of Photographic Magnitudes in Selected Areas 1–139).
P. Stroobant, Cartes célestes destinées spécialement à l'orientation dans les régions équatoriales.

Celestial Photographs.

The following have been added to the list of Celestial Photographs reproduced by the Society for sale to the Fellows :—

R.A.S. Ref. No.
416. Pluto, Lowell Observatory, 42-in. reflector, *F.L.* 220 in., March 2d 4h 56m; March 5d 3h 44m. C. O. Lampland.
417. Solar Corona, 1929 May 9. Takengon, Sumatra, $6\frac{3}{4}$-in. lens, *F.L.* 15 ft. Exp. 70 sec. J. Miller.

Observations of Eros at the Opposition 1930–31. By H. Spencer Jones, Chairman, Solar Parallax Commission, I.A.U.

The approaching opposition of Eros will be widely observed for the determination of the solar parallax, the mass of the moon and related constants. The following remarks are mainly for the general guidance of those who are participating in the photographic observations. In *Ast. Nach.*, No. 5680, directions for the guidance of those undertaking micrometric observations are given by Professors Witt and Kopff.

The motion of Eros near opposition is rapid and mainly in declination ; the maximum motion amounts to nearly 3″ per minute. The

images of either the comparison stars or of Eros must therefore be allowed to trail. Guiding on Eros would have the advantage that the greater error of bisection of a trailed image would be reduced by the number of comparison stars available, but is only possible if the telescope is provided with means for accurate guiding in declination. Where conditions permit, observers should make trial of both methods and use whichever method proves the more convenient and accurate. An alternative method is to guide on Eros in right ascension only ; the image of Eros then would be trailed in the declination direction, and the images of the stars would be trailed slightly in the right ascension direction. Such trails may prove more suitable for accurate bisection than oblique trails. The path of Eros lies in a region which is generally poor in stars ; allowing the stars to trail will restrict to some extent the comparison stars available, and this must be taken into consideration in deciding which method to adopt.

The length of exposure time should be made as short as possible consistent with securing good, measurable images. Observers are recommended to make trials with plates of different speeds : the shorter trails obtained with faster plates as a consequence of the shorter exposures may more than outweigh the disadvantage of coarser grain, and increased accuracy in bisection may result.

Near opposition, Eros is much brighter than the available comparison stars. Its brightness should therefore be reduced by means of a rotating sector, so that the densities of the images of Eros and the comparison stars are reduced to approximate equality ; bisection errors will thereby be reduced and guiding errors eliminated. With the same instrument, the brightness reduction required will depend upon whether the telescope is guided upon Eros or the stars. The brightness of Eros is variable in a period of about five hours, during which time there are two maxima and two minima of light. The range in brightness is not constant but may be as large as 1^m : observers should be on the look-out for such variations and employ the appropriate brightness reduction.

The times of beginning and ending each exposure must be determined with accuracy, in view of the rapid motion of the asteroid. If conditions are not uniform, owing to drifting thin cloud or to an unevenly veiled sky, it would be better not to secure observations, as an appreciable error in the mean time of exposure may be caused.

The parallax of Eros has its maximum value ($50''\cdot3$) about the end of January 1931. Opposition occurs on 17th February. For the derivation of the solar parallax, observations could be restricted to the period when the parallax of Eros is large. For the derivation of the mass of the Moon, however, it is desirable that the observations should be extended over a longer period. Observations for this purpose are required near the times when the lunar equation has its extreme values. In the discussion of the observations secured at the opposition of 1901, it was found that the errors of the ephemeris of Eros could not be represented by an expansion in powers of the time. In order, therefore, to disentangle the errors of the ephemeris from the errors of the lunar equation, arising from the use of an incorrect mass of the Moon, it was

necessary to derive the errors of the ephemeris from observations obtained at the times when the lunar equation had its zero value. From the results the errors at the times when the lunar equation was not zero were derived by interpolation. It is therefore desirable, in order to provide adequate material for the separation of ephemeris errors from errors in the lunar equation, that observations should be obtained not only at times when the lunar equation has its extreme values, but also at the times when it is zero. The lunar equation in R.A. is proportional, *inter alia*, to $\sin(\alpha - \alpha_{\mathbb{C}})$, where α denotes the R.A. of Eros, and $\alpha_{\mathbb{C}}$ the R.A. of the Moon. Observations should therefore be obtained near the times when $\alpha - \alpha_{\mathbb{C}} = 0h, 6h, 12h$, and $18h$, and they should be extended over as many periods of the lunar equation as possible. Observatories which are engaged only in observations by the diurnal method could usefully obtain several plates, with Eros on or near the meridian, at times when the lunar equation is zero or has its extreme values.

The solar parallax can be derived from three types of observation :

(1) From observations taken at a single observatory at large east and west hour angles, *i.e.* evening and morning. Observations should be restricted to altitudes greater than 30° in order to minimise the effects of atmospheric dispersion.

(2) From observations secured simultaneously at two observatories in approximately the same latitude but differing several hours in longitude, *e.g.* near the meridian at the eastern observatory and at large easterly hour angle at the western observatory.

(3) From observations secured simultaneously at two observatories in approximately the same longitude but differing as widely as possible in latitude (*i.e.* a northern and a southern observatory).

Methods (2) and (3) have the advantage of being practically independent of errors in the comparison star-places, as the same comparison stars would be used at the two observatories, and of errors in the ephemeris of Eros. Method (1) is not free from the effects of errors of the star-places, since the motion of Eros is so rapid that the same comparison stars cannot be used for measurement of evening and morning plates secured on the same night. Observations by method (1) can be continued, however, for a longer period than those by methods (2) and (3). Several observatories are co-operating in observations by method (3), and there may be some co-operation by method (2). Observatories desiring to co-operate in observations by methods (2) and (3) should arrange details mutually.

The plates secured by methods (1) and (2) would be measured in right ascension, those by method (3) in declination. Observations by method (3) are possible only during the period when Eros is moving rapidly in declination. The trailed images, whether of Eros or of the stars, can be bisected less accurately when measuring in the declination direction than when measuring in the right ascension direction. In order to minimise errors of bisection, it is suggested that observations by method (3) should be made as follows: Expose for one-quarter of the period required to give good measurable images of the comparison stars, the

telescope being guided on the stars or on Eros in R.A. only (*e.g.* if 2^m exposure is required for comparison stars of $10^m{\cdot}5$–$11^m{\cdot}0$, the exposure would be 30^s). Then close the shutter for a short period (*e.g.* for 2^m). Then give a further exposure of the same duration, followed by another interval, and so on. When four exposures have been given, four short trailed images of Eros will be obtained which are capable of accurate bisection, and the total exposure on the comparison stars will be such as to give good, measurable images. A rotating-sector should be employed, if necessary, with opening adjusted to make the images of Eros and of the comparison stars comparable in density. The measurement being in declination, guiding errors arising from the exposures of Eros and the stars not being synchronous, do not enter.

The primary reference stars, selected by Professor Kopff, are listed in *Ast. Nach.*, Nos. 5375, 5403. The definitive catalogue of the positions of these stars is now in course of preparation at the Astronomisches Recheninstitut. These positions should be used as the basis for all the photographic reductions. For plates obtained with telescopes of the Carte-du-Ciel type, the primary reference stars can be employed as comparison stars. For plates covering smaller fields, obtained with instruments of longer focal length, fainter comparison stars (say of $10^m{\cdot}5$–$11^m{\cdot}0$) must be used. The positions of such stars must be derived photographically, using the primary reference stars as a basis. Arrangements will be made to supply positions of reference stars to any observatory which has not the means to connect the secondary reference stars employed with the adjacent primary reference stars.

With a view to minimising the effects of differential atmospheric dispersion, it is desirable that stars of extreme colour (types B and K5 to M) should not be used as reference stars. The spectral types of the primary reference stars, which are not given in the Henry Draper Catalogue, are listed in *Harvard Bulletin*, No. 871. It is hoped that a survey of secondary reference stars ($10^m{\cdot}5$–$11^m{\cdot}0$) may be made, and the stars of extreme colour listed for exclusion.

Observatories with meridian instruments are requested to obtain as many observations as possible of Eros at meridian transit, in order to control the ephemeris. Several of the primary reference stars should be observed on each night that Eros is observed in order to reduce the observed positions of Eros to the basis of the system of the primary reference stars. In order to eliminate magnitude-error, Eros should preferably be observed through a screen of such density that the magnitude of Eros is brought to equality with the mean magnitude of the primary reference stars.

Royal Observatory, Cape of Good Hope:
1930 *May* 21.

The Orbit of Neptune. By J. Jackson, M.A., D.Sc.

(*Communicated by the Astronomer Royal.*)

Introduction and Summary.—Leverrier's orbit of Neptune was based on two observations by Lalande in 1795 and on a regular series of observations from the time of the discovery of the planet in 1846 to 1873. When this orbit was replaced by Newcomb's in 1904 the planet had fallen about 5″ behind the predicted place. Newcomb used observations till 1898, but his orbit left a residual of 2″ in 1795. Since 1904 the planet has gradually fallen behind Newcomb's prediction, the discordance now amounting to about 3″. The question arises as to how far it is possible to reconcile the observations and theory. This paper shows that by a simple change of the elements not only the modern observations but also Lalande's observations can be satisfied very closely indeed. As only 30 years have to elapse before Neptune returns to the position occupied in 1795, the shape of the orbit should now be satisfactorily known unless, indeed, Lalande's observations were seriously wrong, which can hardly be the case as the two observations show the theoretical motion to a fraction of a second.

Tabular Positions.—In the introduction to his theory Newcomb gives the residuals in longitude and parallax at intervals of 4 years to 1896. The *Nautical Almanac* adopted Newcomb's tables in 1904, and from that date onwards a direct comparison is possible between the observations and theory. To fill the gap, the heliocentric places of Neptune for the first day of each year from 1896 to 1904 were computed. The following table shows the corrections required to reduce Leverrier's tables to Newcomb's.

Newcomb's–Leverrier's Tables.

(January 1 of each year. *N.A.* positions used for Leverrier.)

	$\Delta\lambda$.	$\Delta\beta$.	7th Decimal of log r.
	″	″	
1896	−2·18	−1·78	−51
1897	−2·31	−1·85	−45
1898	−2·76	−1·89	−33
1899	−3·37	−2·02	−26
1900	−3·88	−2·08	−31
1901	−4·33	−2·16	−40
1902	−4·67	−2·15	−51
1903	−4·90	−2·23	−52
1904	−5·06	−2·35	−65

It should be noted that the heliocentric longitude given in the *N.A.* for 1896 refers to the true equinox, but for 1897 and later to the

mean equinox of date, although no mention of change is made in the Introduction. The places in the *Connaissance des Temps* occasionally differ slightly from those in the *N.A.*, although based on the same tables. In the *C.T.* heliocentric positions are given twice, once referred to the true equinox of date and the other time to the mean equinox of 1900·0. The statement in the Introduction to the *C.T.* refers to the latter only. The longitude for 1904 January 1, deduced for this paper from Newcomb's tables, is 0″·14 greater than that given in the *N.A.* from the same tables.

Analysis.—The residuals given by Newcomb up to 1896 and additional ones up to 1928, at 4-year intervals, were then analysed in the form

$$\alpha + \beta t + \gamma \cos g + \delta \sin g,$$

where t is the time in centuries from 1880 and g is the mean anomaly of the planet. The following normal equations were deduced:—

$$\begin{aligned}
22{\cdot}00\alpha &+ 0{\cdot}83\beta &+ 11{\cdot}68\gamma &+ 3{\cdot}57\delta &= &- 16{\cdot}52 \\
+\ 0{\cdot}83\ \alpha &+ 2{\cdot}09\ \beta &+ 0{\cdot}92\ \gamma &+ 3{\cdot}61\ \delta &= &- 3{\cdot}24 \\
+ 11{\cdot}68\ \alpha &+ 0{\cdot}92\ \beta &+ 11{\cdot}30\ \gamma &- 0{\cdot}26\ \delta &= &- 0{\cdot}72 \\
+\ 3{\cdot}57\ \alpha &+ 3{\cdot}61\ \beta &- 0{\cdot}26\ \gamma &+ 10{\cdot}70\ \delta &= &- 12{\cdot}43
\end{aligned}$$

which give

$$\alpha = - 1''{\cdot}60, \quad \beta = - 1''{\cdot}64,$$
$$\gamma = + 1''{\cdot}73, \quad \delta = - 0''{\cdot}02.$$

The correction to the mean motion, − 1″·64 per century, means an increase in the period of about 3 hours. The change in the coefficient of cos g corresponds to a decrease of 36″·7 in the longitude of the perihelion, δ indicates a decrease in the eccentricity of 0″·01.

The following table shows the residuals of Newcomb's orbit which have been analysed, the corrections required by the above change of elements, and the final residuals. The residuals used for the period 1900–1928 were based on the Greenwich and Washington observations.

	O–C (Newcomb).	Corr.	Final Residual.
	″	″	″
1795	−2·05	−1·93	−·12
1848	− ·27	− ·54	+·27
52	+ ·26	− ·25	+·51
56	+ ·07	− ·20	+·27
60	− ·52	− ·07	−·45
64	− ·37	+ ·04	−·41
68	+ ·12	+ ·12	·00
72	− ·30	+ ·16	−·46
76	+ ·20	+ ·16	+·04

	O–C (Newcomb).	Corr.	Final Residual.
	"	"	"
1880	+ ·32	+ ·13	+·19
84	− ·36	+ ·04	−·40
88	− ·30	− ·07	−·23
92	+ ·25	− ·23	+·48
96	− ·28	− ·42	+·14
1900	− ·75	− ·64	−·11
04	− ·72	− ·90	+·18
08	−1·03	−1·19	+·16
12	−1·28	−1·48	+·20
16	−1·71	−1·80	+·09
20	−2·29	−2·13	−·16
24	−2·59	−2·45	−·14
28	−2·92	−2·78	−·14

It is interesting to note that not only are the residuals in Lalande's observations removed, but the average residual between 1848 and 1896 is nearer zero. The corrections to Newcomb's tables for January 1 of the next 20 years are given below.

Corrections to Newcomb's Tables.

1930	−2·94	1940	−3·68
31	−3·02	41	−3·75
32	−3·09	42	−3·81
33	−3·17	43	−3·87
34	−3·25	44	−3·93
35	−3·32	45	−3·99
36	−3·40	46	−4·04
37	−3·47	47	−4·10
38	−3·54	48	−4·16
39	−3·61	49	−4·21
40	−3·68	50	−4·26

Greenwich Observations.—The following table gives the results of the Greenwich observations of Neptune since the opposition of 1896–1897, the last used by Newcomb in the construction of his tables. For the years 1897–1914 the data have been taken from the annual Greenwich volumes, assuming that the earth's orbit is correct as also the radius vector of the planet, any permissible error in which would not appreciably affect the results. For the years 1897–1902 a correction of + 0″·75 has been applied to the observed longitudes to reduce them to Newcomb's equinox. The residuals are in heliocentric longitude ; from

the year 1916 the geocentric longitude has been converted to heliocentric by deducting 1 part in 40.

	Obs.–Lev.	Lev.–Newc.	O–Newc.	Comp. Cor.	Final Residual.	No. of Observations.
1898·0	−2″·85	+2″·77	− ″·08	− ″·53	+ ″·45	28
99·0	−3·46	+3·38	− ·06	− ·59	+ ·53	32
1900·1	−3·65	+3·91	+ ·26	− ·65	+ ·91	16
1·0	−4·23	+4·34	+ ·11	− ·71	+ ·82	18
2·1	−5·09	+4·68	− ·41	− ·77	+ ·36	13
3·1	−5·48	+4·91	− ·57	− ·84	+ ·27	17
4·1	..	..	− ·09	− ·91	+ ·82	19
5·0	..	..	+ ·67	− ·98	+1·65	5
6·1	..	..	+ ·12	−1·05	+1·17	6
7·1	..	..	−1·00	−1·12	+ ·12	14
8·1	..	..	− ·81	−1·19	+ ·38	15
9·1	..	..	− ·66	−1·27	+ ·61	12
10·1	..	..	− ·95	−1·34	+ ·39	21
11·1	..	..	−1·28	−1·42	+ ·14	13
12·1	..	..	−1·00	−1·49	+ ·49	11
13·1	..	..	−1·31	−1·57	+ ·26	7
14·1	..	..	− ·94	−1·65	+ ·71	13
15·0	..	..	−1·54	−1·72	+ ·18	2
16·2	..	..	−1·90	−1·81	− ·09	11
17·2	..	..	−1·74	−1·89	+ ·15	16
18·2	..	..	−2·02	−1·98	− ·04	6
19·2	..	..	−2·03	−2·06	+ ·03	6
20·2	..	..	−2·08	−2·14	+ ·06	14
21·2	..	..	−2·31	−2·22	− ·09	13
22·2	..	..	−1·64	−2·30	+ ·66	3
23·2	..	..	−2·25	−2·38	+ ·13	7
24·2	..	..	−2·60	−2·46	− ·14	9
25·2	..	..	−2·60	−2·54	− ·06	14
26·2	..	..	−2·70	−2·63	− ·07	9
27·2	..	..	−2·58	−2·71	+ ·13	5
28·2	..	..	−3·27	−2·79	− ·48	10
29·2	..	..	−3·11	−2·87	− ·24	11

It should be pointed out that the Washington observations show negative residuals between 1904 and 1914, when the Greenwich residuals are positive.

Comparison with Uranus.—In discussing the residuals of Uranus, without the perturbations of Neptune, Leverrier found the following

residuals after altering the elements to give the best possible fit between 1781 and 1845.

	"		"
1781–1782	+ 20·5	1813–1815	+ 4·5
1783–1784	+ 10·8	1816–1817	+ 6·0
1785–1788	+ 2·0	1818–1820	+ 3·8
1789–1790	− 8·1	1821–1823	+ 1·7
1791–1792	− 7·8	1824–1827	− 7·6
1793–1794	− 10·5	1828–1830	− 7·3
1795–1796	− 10·1	1835	− 4·5
1797–1801	− 6·7	1835–1836	− 4·7
1802–1804	− 3·4	1837–1838	− 2·1
1804–1806	− 0·4	1839–1840	+ 0·7
1807–1808	+ 3·1	1841–1842	+ 1·5
1808–1810	+ 3·8	1842–1844	+ 3·1
1811–1813	+ 4·4	1844–1845	+ 6·5

As it accidentally happens, the relations of Uranus and Neptune in the period 1780–1845 were fairly similar to those of Neptune and the body recently found at Lowell Observatory during the period 1795–1930. Uranus described about three-quarters of its orbit in the interval 1780–1845, Neptune about three-quarters in the interval 1810–1930. Moreover, Uranus and Neptune were in conjunction in 1822, Neptune and the Lowell planet possibly about 1895. These conditions fit so closely as to the time of conjunction and the relative period of observation that we might fairly confidently conclude that there is no body more massive than $\frac{1}{20}$th of Neptune near the position of the new object. The long gap in the observations between 1795 and 1846 is the main reason why we have not suggested a still smaller maximum mass. It may be worth pointing out that the existence of a disturbing body would be shown by the residuals running through two complete cycles in the interval covered by the observations. When we analyse the residuals in longitude to correct the elements we really have four disposable constants, and the natural effect of attempting to fit the observed and computed longitudes (if a disturbing body is present) is to make the residuals vanish four times in the interval covered by the observations. This is very clearly shown in the residuals Leverrier found after making the best fit for Uranus, and the fact that a similar series of residuals is not found in the case of Neptune indicates that the effect of such outside bodies as may exist does not exceed the error of observation. The period shown by the residuals should be about half that of the interval covered by the observations and has no relation to the period of the actual perturbations. The mass of the newly found body may therefore be taken as less than that of the earth, and not greater than would be obtained by giving it the lowest albedo and the highest density of the major planets. Once its orbit is more definitely known it will be possible to deduce its mass from the perturbations of Neptune, but it seems unlikely that it will be larger than its probable error.

A New Determination of the Constant of Nutation.

By J. Jackson, M.A., D.Sc.

(*Communicated by the Astronomer Royal.*)

Introduction.—The determination of the constant of nutation presents no serious practical difficulties beyond those associated with making a series of observations of the same stars on a uniform system extending over an interval of two decades. The principal term of the nutation, to the coefficient of which the term "constant of nutation" is given, has a period of 18·6 years, and the positions of stars relative to the true equator and equinox show displacements in both coordinates in this period.

As the displacement is periodic it can be separated from constant terms and secular terms, and so the result is not affected by systematic or peculiar motions of the stars observed. It is thus free from the principal difficulties attending the determination of the constant of precession. As the period of the principal term is many years, the same star is observed at the same season of the year throughout the period and the observations are free from systematic effects depending on seasonal changes, such as make the determination of the aberration constant so difficult.

The Cookson floating telescope has now been in use at Greenwich for 19 years, and as about two-thirds of the stars of the original observing programme are still being observed it appeared that the observations would be very suitable for the determination of the constant of nutation.

As is well known, the Cookson telescope has been employed at Greenwich principally for the determination of latitude variation. The telescope is mounted on a float, and the method of observing is that of photographing the trails of pairs of stars which transit the meridian at nearly equal distances north and south of the zenith. The distance between two star trails is twice their mean zenith distance.

The constant of nutation used in ephemerides was derived by Newcomb principally from observations made between 1850 and 1895 at Greenwich, Pulkovo, and Washington, but also extending backwards to observations by Bradley with the zenith sector. Both right ascensions and declinations were used. The resultant value was $9''{\cdot}210 \pm ''{\cdot}008$. This has been closely confirmed by Przybyllok,* who obtained $9''{\cdot}207 \pm ''{\cdot}003$ from 15 years' observations at five stations of the International Latitude service. The latter result is in weight superior to all previous determinations combined, although it would have been much improved if the observations had extended a few years longer.

* "Zentralbureau der Internationalen Erdmessung," *Neue Folge der Veröffentlichungen*, Nr. 36, 1920.

The principal terms in the nutation for which it is desired to improve the coefficients are

$$\text{Nutation in longitude} = \Delta\psi = (-17''{\cdot}234 - ''{\cdot}017\mathrm{T}) \sin\Omega$$
$$\text{,, in obliquity} = \Delta\epsilon = (+9''{\cdot}210 + ''{\cdot}0009\mathrm{T}) \cos\Omega$$

where T is counted in centuries from 1900 and Ω is the longitude of the moon's node.

The effect of the nutation on declination is

$$\Delta\delta = \cos\alpha \sin\epsilon\, \Delta\psi + \sin\alpha\, \Delta\epsilon,$$

and the observations give

$$\text{zenith distance north} = \delta - \phi.$$

Now δ can be put in the form $\delta = \delta_0 + \mu t +$ periodic terms, the periodic terms including the nutation. The solution would be much simpler if the latitude ϕ could be treated as constant or known, as then each observed zenith distance would give an equation for the determination of the constant of nutation. But as ϕ is variable in an irregular manner to the extent of $\pm\, 0''{\cdot}3$, this cannot be done for observations at a single station.

In the case of the international stations, which were five in number and well separated in longitude, the value of ϕ was taken as

$$\phi = \phi_0 + x \cos\lambda + y \sin\lambda,$$

where λ is the longitude and x, y were assumed to be known. It is clear that if the assumed values of x, y have a component of period 18·6 years they will affect the value of the constant derived at the various stations, but that an analysis of the constant derived from the separate stations will enable this effect to be eliminated. The only remaining uncertainty lies in the existence of the z term in the latitude variation, which would affect all stations alike.

At the international stations the observations are made round about midnight. At Greenwich, in order to make the observations suitable for the determination of aberration and to give a strong link between the final values for the mean declinations of the stars in different right ascensions, the observations are made after sunset and before sunrise. This means that stars of quite different right ascensions were observed on the same nights so that the difference of observed zenith distances in different R.A.'s could be used to determine the constant of nutation independently of the latitude. The strength of this method lies in the complete elimination of uncertainties arising from latitude variation, but its weakness lies in the fact that for a considerable portion of the year the range in right ascensions is small, and indeed for the summer months no results at all can be obtained. It might seem that the latitude variation having been determined it might be used, and so enable results of greater weight to be obtained. But the assumption of a knowledge of the latitude variation, whether made at Greenwich or not, would involve an assumption of the constant of nutation. Latitude variation and nutation can only be separated by observations in different right ascensions or in different longitudes.

The Reductions.—The observing programme with the Cookson telescope at Greenwich consists of 16 groups, each with 4 or 5 pairs of stars. The following tables give the approximate R.A. of the groups and the periods of observation :—

Group.	R.A. h h	Group.	R.A. h h
1	0 – 1	9	14½–16
2	2½– 3½	10	16 –17
3	3½– 4½	11	17 –18
4	7½– 8½	12	18 –19
5	8½– 9½	13	19 –20
6	11 –12	14	20 –21
7	12 –13	15	21 –22
8	13½–14½	16	23 –24

Period.	Evening Group.	Morning Group.
Dec. 25–Jan. 31	2, 3	6, 7
Feb. 14–Mar. 23	4, 5	8, 9
Mar. 24–Apr. 26	6, 7	9, 10
Apr. 27–May 25	8, 9	10, 11
May 30–June 21	10, 11, 12	
June 22–July 9	11, 12, 13	
July 14–Aug. 7	12, 13	14, 15
Aug. 12–Sept. 19	13, 14	16, 1
Sept. 20–Oct. 25	14, 15	2, 3
Nov. 11–Dec. 22	16, 1	4, 5

During the short gaps in the above schedule and occasionally at other times scale plates were taken.

The observations made with the Cookson telescope for the years 1911–1919 and 1919–1927 are given in two small volumes published by the Royal Observatory. Additional observations up to the end of 1929 have been used in the following discussion. The standard system of star places and proper motions on which this discussion rests is given on pp. 24 and 25 of the volume giving the 1919–1927 observations. The mean places published in the two volumes mentioned above have been corrected with the constants given in the *Nautical Almanac*, including the short-period lunar terms, but without proper motion.

The observed data were therefore first of all corrected for proper motion, for scale value, and for temperature, according to the data given in the volumes. For a discussion of the constant of nutation it is necessary to apply other corrections, which may be safely neglected in latitude variation but which, from the fact that they have a long period or a period not differing very much from a year, may not be safely neglected in this work. In the first place, the aberration applied in the *N.A.* formulæ does not take account of the perturbations by Jupiter and

Saturn which have periods of about 12 and 30 years respectively. These were applied in the form given by Battermann,*

$$C[\cos\lambda(\cos\epsilon\sin\delta\sin\alpha - \sin\epsilon\cos\delta) - \sin\lambda\sin\delta\cos\alpha],$$

where λ is the heliocentric longitude of the planet and $C = 0''{\cdot}0086$ for Jupiter and $C = 0''{\cdot}0019$ for Saturn. The maximum value of these corrections was about $0''{\cdot}010$. In addition, Dr. F. E. Ross † showed that the following small terms in the nutation, generally neglected, should be applied:

$$\begin{aligned}\sin\epsilon\delta\lambda = &+ {}''{\cdot}0012\sin 2\Omega\\ &- {\cdot}0006\sin 2\omega\\ &+ {\cdot}0021\sin(2\omega+\Omega)\\ &+ {\cdot}0002\sin(2\omega+2\Omega)\\ &+ {\cdot}0049\sin(2g'+2\omega'+\Omega)\\ &+ {\cdot}0018\sin(2g'+2\omega'-2\omega)\\ &- {\cdot}0008\sin(2g'+2\omega')\\ &+ {\cdot}0005\sin(g'-\Omega)\\ &- {\cdot}0006\sin(g'+\Omega)\end{aligned}$$

$$\begin{aligned}\delta\epsilon = &- {}''{\cdot}0007\cos 2\Omega\\ &- {\cdot}0028\cos(2\omega+\Omega)\\ &- {\cdot}0066\cos(2g'+2\omega'+\Omega)\\ &+ {\cdot}0006\cos(g'-\Omega)\\ &+ {\cdot}0008\cos(g'+\Omega)\end{aligned}$$

where unaccented symbols refer to the elements of the moon's orbit and accented to those of the sun's orbit.

The values of these quantities for the interval 1900–1920 were given by Ross. Tables I. and II. give their values for the additional years covered by the present investigation, and, like the Tables II*a* and II*b* given by Ross, include the terms depending only on the sun's longitude, tabulated by him as Table I.

TABLE I.

Values of sin εδλ neglected in National Ephemerides. Unit ″·001.

Fraction of Year.	1920.	1921.	1922.	1923.	1924.	1925.	1926.	1927.	1928.	1929.
·00	−1	0	+ 3	+ 2	+ 4	+ 7	+3	+ 1	+ 6	+7
·03	+2	+ 4	+ 6	+ 4	+ 6	+ 9	+4	+ 1	+ 5	+7
·06	+5	+ 8	+ 9	+ 6	+ 7	+10	+5	0	+ 4	+6
·09	+7	+10	+11	+ 6	+ 6	+ 9	+4	− 2	+ 2	+4
·12	+7	+11	+10	+ 4	+ 3	+ 6	+1	− 5	− 1	+2
·15	+5	+ 9	+ 8	0	− 1	+ 2	−2	− 8	− 4	−1
·18	+3	+ 7	+ 6	− 4	− 5	− 2	−5	−10	− 7	−4
·21	0	+ 4	+ 3	− 7	− 9	− 5	−7	−11	− 8	−5
·24	−2	+ 1	0	−10	−12	− 8	−9	−11	− 8	−5
·27	−4	0	− 1	−10	−12	− 8	−7	− 8	− 5	−3
·30	−5	− 2	− 2	− 9	−10	− 6	−5	− 4	− 1	0
·33	−6	− 2	− 2	− 8	− 8	− 4	−3	0	+ 3	+3

* "Resultate aus den Polhohenbestimmungen in Berlin," *Centralbureau der Internationalen Erdmessung*, 1899.

† *Astr. Nach.*, **192**, 47, 1912.

TABLE I.—*continued.*

Fraction of Year.	1920.	1921.	1922.	1923.	1924.	1925.	1926.	1927.	1928.	1929.
·36	−6	− 3	− 1	− 5	− 4	− 1	0	+ 4	+ 7	+6
·39	−6	− 3	− 1	− 3	− 2	+ 2	+3	+ 6	+10	+8
·42	−6	− 3	0	− 2	+ 1	+ 4	+3	+ 7	+11	+8
·45	−6	− 3	0	0	+ 3	+ 5	+3	+ 6	+10	+7
·48	−6	− 2	0	+ 1	+ 5	+ 6	+2	+ 5	+ 9	+5
·51	−4	0	+ 1	+ 1	+ 6	+ 6	+1	+ 2	+ 7	+3
·54	−2	+ 2	+ 1	+ 1	+ 6	+ 5	0	0	+ 4	+2
·57	0	+ 4	+ 2	0	+ 5	+ 4	−3	− 3	+ 2	0
·60	+2	+ 6	+ 3	0	+ 4	+ 3	−4	− 5	0	−1
·63	+5	+ 9	+ 4	0	+ 3	+ 3	−4	− 5	0	−1
·66	+7	+10	+ 5	− 1	+ 2	+ 2	−4	− 5	0	0
·69	+8	+11	+ 4	− 2	+ 1	+ 2	−4	− 4	0	+1
·72	+7	+10	+ 3	− 4	− 1	+ 1	−3	− 3	+ 1	+2
·75	+5	+ 8	+ 2	− 6	− 3	− 1	−3	− 2	+ 2	+3
·78	+2	+ 5	− 1	− 7	− 5	− 2	−3	− 1	+ 2	+3
·81	−2	+ 2	− 3	− 9	− 6	− 3	−3	0	+ 3	+2
·84	−5	− 2	− 5	− 9	− 6	− 4	−3	+ 1	+ 3	+2
·87	−8	− 4	− 6	− 9	− 6	− 4	−2	+ 2	+ 3	+1
·90	−9	− 5	− 6	− 7	− 4	− 3	−2	+ 3	+ 4	0
·93	−8	− 4	− 4	− 4	− 1	− 1	−1	+ 4	+ 5	0
·96	−5	− 1	− 2	0	+ 3	+ 1	0	+ 6	+ 6	+1
·99	−1	+ 2	+ 1	+ 3	+ 6	+ 3	+1	+ 6	+ 7	+1
1·02	+3	+ 5	+ 4	+ 5	+ 8	+ 4	+1	+ 6	+ 7	+1

TABLE II.

Values of δε neglected in the National Ephemerides. Unit ″·001.

Fraction of Year.	1920.	1921.	1922.	1923.	1924.	1925.	1926.	1927.	1928.	1929.
·00	−10	−15	−12	−10	−12	−10	− 3	− 1	− 2	+ 2
·03	−10	−13	−10	− 7	− 9	− 6	+ 1	+ 3	+ 1	+ 5
·06	− 8	−10	− 6	− 2	− 4	− 1	+ 6	+ 6	+ 4	+ 7
·09	− 5	− 7	− 1	+ 2	+ 1	+ 3	+10	+ 9	+ 6	+10
·12	− 2	− 3	+ 3	+ 6	+ 4	+ 6	+12	+10	+ 6	+ 8
·15	+ 1	0	+ 6	+ 8	+ 6	+ 8	+12	+ 9	+ 5	+ 6
·18	+ 3	+ 2	+ 8	+ 9	+ 6	+ 7	+11	+ 7	+ 2	+ 3
·21	+ 4	+ 3	+ 8	+ 9	+ 4	+ 5	+ 8	+ 3	− 2	0
·24	+ 5	+ 4	+ 8	+ 7	+ 2	+ 2	+ 4	0	− 5	− 4
·27	+ 4	+ 2	+ 6	+ 4	− 1	− 2	0	− 5	− 9	− 6
·30	+ 3	+ 1	+ 3	+ 1	− 5	− 5	− 3	− 8	−12	− 8
·33	+ 2	0	+ 2	− 1	− 7	− 7	− 5	− 9	−12	− 8

TABLE II.—*continued.*

Fraction of Year.	1920.	1921.	1922.	1923.	1924.	1925.	1926.	1927.	1928.	1929.
·36	+ 2	− 2	0	− 2	− 8	− 7	− 5	− 8	−10	− 5
·39	0	− 3	− 1	− 4	− 9	− 7	− 4	− 7	− 8	− 2
·42	− 1	− 4	− 2	− 4	− 8	− 5	− 2	− 4	− 4	+ 2
·45	− 2	− 4	− 2	− 4	− 7	− 3	+ 1	− 1	0	+ 5
·48	− 4	− 5	− 1	− 2	− 5	0	+ 4	+ 2	+ 3	+ 8
·51	− 6	− 6	− 1	− 2	− 4	+ 2	+ 6	+ 4	+ 4	+ 9
·54	− 8	− 6	− 1	− 2	− 3	+ 3	+ 7	+ 4	+ 4	+ 8
·57	− 8	− 6	0	0	− 1	+ 5	+ 8	+ 4	+ 4	+ 7
·60	− 8	− 5	+ 1	+ 1	0	+ 6	+ 8	+ 4	+ 3	+ 5
·63	− 6	− 3	+ 3	+ 2	+ 1	+ 6	+ 8	+ 2	+ 1	+ 2
·66	− 4	− 1	+ 5	+ 3	+ 2	+ 6	+ 6	0	− 2	0
·69	− 1	+ 2	+ 7	+ 4	+ 2	+ 6	+ 5	− 1	− 3	− 2
·72	+ 2	+ 5	+ 9	+ 5	+ 3	+ 5	+ 4	− 2	− 4	− 2
·75	+ 5	+ 7	+10	+ 6	+ 3	+ 5	+ 4	− 3	− 4	− 2
·78	+ 6	+ 8	+10	+ 5	+ 2	+ 4	+ 2	− 3	− 3	− 1
·81	+ 6	+ 7	+ 8	+ 2	− 1	+ 2	+ 1	− 4	− 3	0
·84	+ 3	+ 4	+ 4	− 1	− 4	− 1	− 1	− 6	− 4	+ 1
·87	0	0	0	− 5	− 7	− 3	− 3	− 6	− 3	+ 2
·90	− 4	− 4	− 4	− 9	−10	− 5	− 4	− 6	− 3	+ 3
·93	− 9	− 8	− 7	−11	−11	− 6	− 4	− 6	− 2	+ 4
·96	−13	−11	−10	−13	−12	− 6	− 4	− 5	0	+ 4
·99	−15	−12	−10	−13	−11	− 4	− 2	− 4	+ 1	+ 5
1·02	−14	−11	− 8	−10	− 8	− 1	+ 1	0	+ 4	+ 6

The Analysis.—The residuals so corrected were then collected. For each tenth of a year the mean of the observations for each pair was taken and then the mean for all the evening and all the morning pairs. The following table gives the data for the first epoch 1911·7 :—

Evening Observations.

Group.	Pair.	No. of Observations.	Residual.	Nutation Factor.
			″	
13	3, 4	2	−·140	+0·83
	5, 6	2	−·537	+ ·85
	7, 8	3	−·082	+ ·85
	9, 10	3	−·329	+ ·85
14	1, 2	3	−·003	+ ·84
	3, 4	3	−·324	+ ·83
	5, 6	2	+·078	+ ·84
	9, 10	2	−·018	+ ·84
		Mean	−·169	+0·84

Morning Observations.

Group.	Pair.	No. of Observations.	Residual.	Nutation Factor.
			″	
16	1, 2	3	−·208	+0·58
	5, 6	3	−·261	+ ·49
	7, 8	3	−·183	+ ·45
1	3, 4	3	−·218	+ ·35
	5, 6	3	+·034	+ ·20
	7, 8	3	−·218	+ ·28
		Mean	−·176	+0·41

In some cases the number of times each pair was observed was rather irregular, but means were taken independently of the number of observations. The nutation factor was computed in the form

$$N = +\,0{\cdot}744 \cos a \sin \Omega \;-\; 1{\cdot}000 \sin a \cos \Omega$$
$$-\,0{\cdot}119 \cos a \sin 2\odot + 0{\cdot}129 \sin a \cos 2\odot,$$

thus including the principal solar as well as the principal lunar terms.

TABLE III.

Mean Residuals for Groups 14, 15 *observed in the evening, and Groups* 2, 3 *observed in the morning at same time of year, and corresponding Nutation Factors.*

	Residuals.			Nutation Factors.			No. of Observations.	
	Evening.	Morning.	Diff.	Evening.	Morning.	Diff.	Evening.	Morning.
	″	″	″					
1911	−·127	−·059	−·068	+·75	−·35	+1·10	33	11
12	+·159	+·187	−·028	+·68	−·63	+1·31	27	24
13	+·130	+·157	−·027	+·51	−·74	+1·25	45	12
14	−·023	+·061	−·084	+·30	−·82	+1·12	34	20
15	−·038	−·189	+·151	−·01	−·81	+0·80	41	26
16	−·293	−·188	−·105	−·29	−·72	+0·43	60	30
17	−·183	−·277	+·094	−·56	−·48	−0·08	52	19
18	−·056	−·125	+·069	−·78	−·20	−0·58	64	14
19	+·032	+·061	−·029	−·94	+·08	−1·02	57	26
20	−·107	−·049	−·058	−·99	+·34	−1·33	69	28
21	−·069	−·142	+·073	−·95	+·62	−1·57	61	50
22	−·100	−·142	+·042	−·81	+·79	−1·60	66	38
23	−·106	−·140	+·034	−·59	+·91	−1·50	63	23
24	−·017	·000	−·017	−·32	+·88	−1·20	51	25
25	+·019	−·058	+·077	−·03	+·78	−0·81	34	27
26	−·064	−·064	·000	+·25	+·63	−0·38	32	26
27	−·047	−·069	+·022	+·49	+·41	+0·08	71	19
28	−·008	+·004	−·012	+·66	+·05	+0·61	58	45
29	+·021	−·020	+·041	+·75	−·23	+0·98	38	28

The differences between the evening and morning observations were next taken for each year, as shown in Table III. for evening groups 14, 15 and morning groups 2, 3.

The differences between the evening and morning residuals were analysed in the form $\alpha + \beta t + \gamma N$, where γ is the correction to the nutation constant. To begin with, no attention was paid to the number of observations in each year, and the residuals were analysed from normal equations formed without weighting the results for separate years. When α and β were eliminated from the normal equations the following results were obtained :—

Groups.		Weight from No. of Observations.
14, 15 ; 2, 3	$16{\cdot}69\gamma = -''{\cdot}183$	4·1
16, 1 ; 4, 5	$18{\cdot}34 \quad +{\cdot}092$	2·2
2, 3 ; 6, 7	$8{\cdot}26 \quad +{\cdot}096$	2·2
4, 5 ; 8, 9	$10{\cdot}91 \quad -{\cdot}118$	2·3
6, 7 ; 9, 10	$5{\cdot}18 \quad +{\cdot}003$	2·3
8, 9 ; 10, 11	$2{\cdot}75 \quad +{\cdot}059$	3·1
12, 13 ; 14, 15	$0{\cdot}57 \quad +{\cdot}037$	3·8
13, 14 ; 16, 1	$4{\cdot}82 \quad -{\cdot}093$	3·9

For each of the equations the weight is that of the coefficient of γ (which depends on the interval between the evening and morning groups) multiplied by the weight derived from the number of observations. Unit weight corresponds to 8 evening and 8 morning pairs, so that the probable error corresponding to unit weight is that of a plate. We thus get on multiplying the equations by the weight depending on number of observations,

$68{\cdot}4\gamma$	=	$-''{\cdot}750$
$40{\cdot}3$		$+{\cdot}202$
$18{\cdot}2$		$+{\cdot}211$
$25{\cdot}1$		$-{\cdot}271$
$11{\cdot}9$		$+{\cdot}007$
$8{\cdot}5$		$+{\cdot}183$
$2{\cdot}2$		$+{\cdot}141$
$18{\cdot}8$		$-{\cdot}363$
$193{\cdot}4$		$-{\cdot}640$

whence $\gamma = -''{\cdot}0033.$

On account of the considerable variation in the number of observations made each year it was decided to make a duplicate solution in which the results for each year were weighted according to the formula $\frac{a \times b}{a + b}$, where a, b are the number of evening and morning observations.

The solution carried out in this way gives the following results from the different groups :—

$76 \cdot 8\gamma$	=	$-''\cdot 858$
35·9		$+ \cdot 420$
19·7		$+ \cdot 215$
27·5		$- \cdot 375$
10·4		$- \cdot 007$
8·6		$+ \cdot 105$
2·4		$+ \cdot 092$
22·8		$- \cdot 290$
204·1		$- \cdot 698$

whence $\gamma = -''\cdot 0034$.

There are several methods of computing the probable error of the result. If the differences between the values obtained for the various groups are used, the probable error is $\pm''\cdot 0033$ for the unweighted solution and $\pm''\cdot 0030$ for the weighted one. These probable errors appear to be much too small, and in fact must be considered rather unreliable on account of the smallness of the number of determinations and the great range in their weights. A more reliable determination is by means of the separate observations. The probable error of a plate deduced in the original discussion of the observations was $\pm''\cdot 078$, from which we would deduce the probable error of the nutation constant as $\pm''\cdot 0055$. Even by using the probable error of a single plate as $\pm''\cdot 059$, which was obtained by neglecting the " error of the night " (an error which can be only partly eliminated by the use of the difference of the morning and evening observations), the probable error of the result would still be $\pm''\cdot 0042$.

The probable error can also be determined from the residuals of the equations of condition. From a discussion of some of these the larger probable error is indicated. Hence we write as our final result for the constant of nutation

$$9''\cdot 2066 \pm''\cdot 0055.$$

It is remarkable how the separate determinations of this constant agree so well within their probable errors—an indication of the anticipated freedom of the results from systematic error. From all the determinations the value may be taken as

$$9''\cdot 2075 \pm''\cdot 002,$$

agreeing with that adopted by de Sitter (*B.A.N.*, No. 129, vol. 4, p. 57, 1927).

Relation to other Fundamental Constants. — It is well known that the constants of precession and nutation depend on the function $\frac{C - A}{C}$ of the moments of inertia of the earth and on μ, the mass of the moon. If the two constants have been determined from observation, then both

$\frac{C-A}{C}$ and μ can be calculated. However, these quantities are not easily separated, as nutation involves both as factors, and the larger part of precession also involves both as factors. The mass of the moon can be determined from the displacement of the earth from the centre of the earth-moon system, as shown by observations of the sun, or better of minor planets during close approaches to the earth. Consequently, if the mass of the moon is accepted from the latter observations, it will enable us to calculate the constant of nutation. Newcomb, as we have already stated, determined the constant of nutation from observations up to 1895 as 9″·210, but at the Paris conference of 1896 he urged the adoption of 9″·214 derived from the constant of precession and the mass of the moon. It has since been shown that by the general theory of relativity, the orbital motion of the earth contributes 1″·91 per century to precession. When this is subtracted from the portion originally ascribed to the attraction of the sun and moon the theoretical value of the constant of nutation is reduced from 9″·214 to 9″·210, agreeing exactly with Newcomb's value.* The values obtained both by Przybyllok and in this paper are somewhat smaller still, viz. 9″·207. Oort † finds from a determination of galactic rotation that Newcomb's precession should be increased by 1″·13 per century. If this were accepted, the discordance between the observed and computed constants of nutation would be increased by 0″·002.

Addendum, 1930 *June* 30.

Since the above was written Dr. Fotheringham has pointed out to me that the relativity effect on precession has previously been applied with the wrong sign. I have consulted Sir Arthur Eddington and Professor de Sitter on this question and they both agree that this is the case. Consequently, the theoretical value of the constant of nutation based on Newcomb's value of the precession and mass of the moon is 9″·218 and not 9″·210. If Oort's correction to Newcomb's precession is also applied, the theoretical value of the constant of nutation is still further raised to 9″·220. The observed value, as we have already seen, differs from this by about six times its probable error. This must now be considered as one of the outstanding discordances of the constants of the solar system. It may be that the correction of the precession by 1″·91 per century is not the only change in the relationship between the constants which results from the theory of relativity. The question has not yet been completely investigated theoretically, and the discordance we have found points to the need of further theoretical investigations. The approaching opposition of Eros will give, especially to observers in the southern hemisphere, an ideal opportunity to redetermine the mass of the moon. In the determination of the constant of nutation the results of greatest weight have been those from zenith distance observations. It may soon be possible to redetermine it from series of observations in R.A. made with the self-registering micrometer.

* But see addendum.

† *B.A.N.*, No. 132, **4**, 86, 1927.

Star-streaming in Relation to Spectral Type.

By W. M. Smart, M.A., D.Sc.

(*Communicated by the Director, Cambridge Observatory.*)

1. In *M.N.*, **87**, 122, 1926, I gave the analysis,* for star-streaming, of the photographic proper motions of 3029 stars mainly between photographic magnitudes 9 and 12. The stars were divided into eight groups according to their positions in the sky, and the calculation of the constants of star-streaming was made from the distribution of the proper motions in these eight composite regions.

It became a matter of considerable interest to investigate the contribution of the various spectral types to the general phenomenon of star-streaming. In the past this has been done by several investigators for the bright stars whose proper motions have been mainly determined from meridian-circle observations. In particular, it has generally been found that the B-type stars share to a very small extent, if at all, in the usual streaming shown by the other spectral types.

It was proposed to divide the total number of stars available in any single group into three sub-groups according to colour-index, and to investigate the distribution of the proper motions in these sub-groups individually. Two requirements are essential; the total number of stars in the group must be sufficiently large to give an adequate number of stars in each of the sub-groups, and the position angles of Drift I. and Drift II. must be separated as widely as possible. With these considerations in view, the stars of Group I.† (consisting of the regions of λ And., Groombridge 34, η Cas.) were selected. To these were added the stars (121 in number) of the region of β Cas., whose proper motions had recently been measured. Table I. summarises the relevant information for the amplified Group I.

Table I.

Number of proper motions	666
Centre of region	$0^h\ 14^m$, $+52°$
Position-angle of Drift I.	103°
Position-angle of Drift II.	200°

2. Dr. Knox-Shaw very kindly undertook to photograph the four regions, mentioned in the previous section, at the Radcliffe Observatory, Oxford. The plates used were Ilford Panchromatic; four exposures (denoted by A, B, C, D) were made through a Wratten blue filter and two (denoted by P, Q) through a yellow filter. A suitable adjustment for focus was made separately for the two screens. The details are as follows:—

Table II.

Blue Screen Exposures.				Yellow Screen Exposures.	
A.	B.	C.	D.	P.	Q.
min.	min.	sec.	sec.	min.	min.
6	2	30	10	30	$2\frac{1}{2}$

* See also *Cambridge Astronomical Observations*, **26** (1928).

† *M.N.*, **87**, 123 (Table I.), 1926.

The diameters of the stellar images were measured with the Cambridge measuring machine by Mr. and Mrs. H. E. Green.

The first procedure was to derive mean curves giving the relation between diameters and the logarithms of exposure-ratios for the images through the blue screen and through the yellow screen. Table III. gives the mean values of the diameters for groups of stars in the field of λ Andromedæ. The diameters are expressed in terms of the micrometer unit of the measuring machine.

Table III.

Diameter Measures for the Region of λ And.

No. of Stars.	Exposures.				No. of Stars.	Exposures.	
	A.	B.	C.	D.		P.	Q.
3	537	406	274	192	4	418	193
4	393	299	177	107	4	345	156
7	353	262	154	100	10	304	131
8	299	210	113	70	10	248	100
18	205	144	86	..	10	206	83
23	151	106	..	..			

The corresponding curves are shown in fig. 1, where the abscissæ are the logarithms of the exposure-ratios (multiplied by a suitable scale-factor); in particular, the abscissa corresponding to the exposures A and B is proportional to the logarithm of the ratio 6 : 2, the abscissa corresponding to the exposures A and C is proportional to the logarithm of the ratio 6 : $\frac{1}{2}$, and so on.

The exposure-ratios between A and C and between P and Q are the same; the curves between A and C on the left-hand side of fig. 1 correspond very closely to the PQ curves when the diameters of A are equal to the diameters of P; this is evidently true also for the corresponding interpolated curves.

For bright stars of spectral type about G5–K0 (see Table IV., which refers to the λ And. region), the measures of the A and P diameters are approximately the same; the measures of the Q diameters are a little less than those of the C diameters.

3. The criterion for the colour-indices of the individual stars was applied as follows. For a particular star the measure of the A diameter enables us to interpolate a diameter-exposure-ratio curve in fig. 1. On this curve the measure of the P diameter indicates the length of the exposure through the blue filter, which would yield the same size of image as that of the P exposure. If ΔL denotes the logarithm of the ratio of this exposure to that of A, the values of ΔL are negative for stars of type earlier than about G5–K0 and positive for later types. (The details mentioned here apply only to the λ And. region.)

Consider now the diameter of the star for the B exposure. Through the corresponding point in fig. 1 we can interpolate a diameter-exposure-

ratio curve, and using the value of the P diameter we obtain a second value of ΔL. The process is repeated for the C and D diameters. It is evident that exposures C, D, and Q can be treated in a similar way.

For a star with measurable images in all six exposures there are thus six deducible values of ΔL. These were suitably weighted and the mean taken. For about 50 per cent. of the stars the images for the

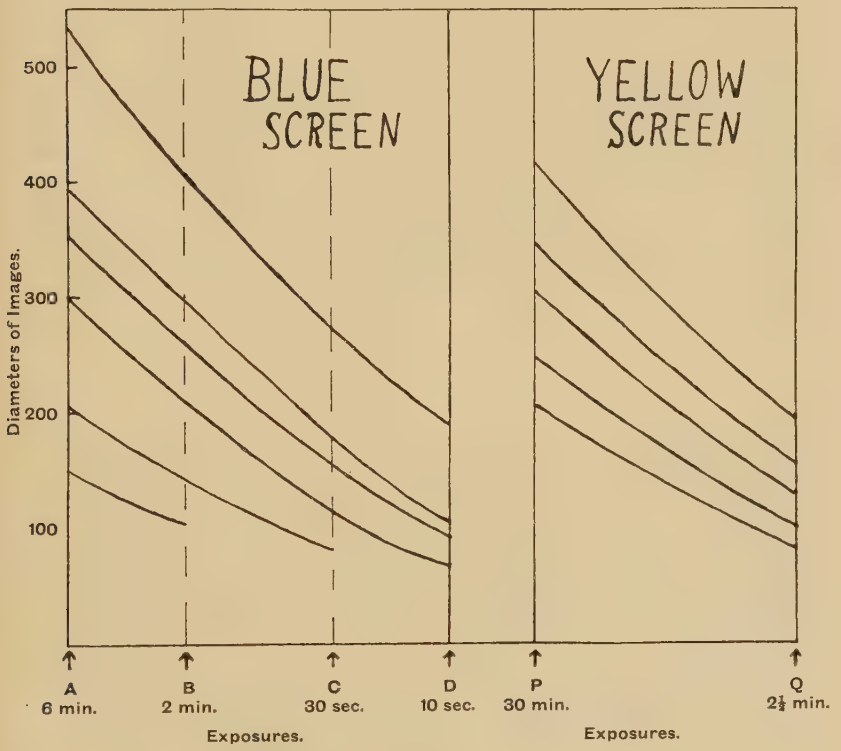

FIG. 1.

C, D, and Q exposures were absent; thus the values of the quantity ΔL for one of these stars depends on only two determinations, which, of course, are not independent, since the measure of the P diameter enters into each.

4. The relation of the quantity ΔL to spectral type can only be determined from the measures of stars whose types are known. Unfortunately in each of the four regions there are only half a dozen stars, on the average, which appear in the Harvard Catalogue, and these are all bright stars. Table IV. gives the relevant information for the five Harvard stars in the region of λ Andromedæ (λ And. itself is omitted as it is much too bright to be used profitably in this discussion).

TABLE IV.

Values of ΔL for H.D. Stars in the Region of λ And.

H.D. No.	Mag.		Spectrum.	Diameter of Image.						ΔL_1.	ΔL_2.
	Pv	*Pg*		A.	B.	C.	D.	P.	Q.		
222000	9·0	9·0	A0	396	290	178	107	337	164	−0·56	−0·20
64	8·0	8·0	A0	506	360	234	162	433	201	−0·42	−0·40
132	8·0	8·0	A0	515	391	271	176	437	217	−0·60	−0·56
143	6·6	7·3	G5	696	550	392	280	672	368	−0·14	−0·24
30	8·6	9·6	K0	414	312	205	106	408	186	−0·06	−0·16

In the penultimate column ΔL_1 is the value of ΔL when the measure of the P diameter is used in conjunction with the curve given by the diameters of the exposures A to D, and ΔL_2 is the value of ΔL when the measure of the Q diameter is used in conjunction with the curve given by the C and D exposures. The values of ΔL_1 and ΔL_2 contain the arbitrary scale-factor previously alluded to. Summarising the results in Table IV. we have the following mean values of ΔL (ΔL_1 and ΔL_2 are given the weights 2 and 1 respectively) for the available stars of the region.

TABLE V.

Spectrum.	A0.	G5.	K0.
ΔL	−0·48	−0·17	−0·09

So far as it goes, the relation between ΔL and spectral type given by the figures of Table V. is almost a linear one.

The stars belonging to the region of λ And. were then divided into three nearly equal sub-groups according to the values of ΔL. Using the relation just referred to and extrapolating where necessary, it was found that the sub-groups could be classified as regards spectral type as follows:—(i) B–G2; (ii) G3–K7; (iii) K8–M. For convenience we call the stars of the sub-groups white stars, intermediate stars, and red stars.

Taking all four regions together, we have the broad classification:

White stars	Types B, A, F.
Intermediate stars	Types G–K5.
Red stars	Types K5–M.

5. We consider now the contribution to star-streaming of each of the three colour-groups of stars.

As the Radcliffe telescope has a somewhat smaller field than the Sheepshanks telescope (with which the proper motion plates were taken), the total number of stars in the four regions with measured colour characteristics is just three-quarters of the number for which proper motions are known (the numbers are 500 and 666 respectively). The three colour groups contain respectively 159, 174, and 167 stars. Table VI. gives the details of the distribution of the proper motions in position angle for (*a*) all 666 stars, (*b*) 159 white stars, (*c*) 174 intermediate stars, and (*d*) 167 red stars. The counts were made first in

10° sectors of position angle and smoothed by taking the sums of the counts in three adjacent sectors. These sums are given in the table.

TABLE VI.

Distribution of Proper Motions in Position Angle.

Position Angle.	All Types. (*a*).	White Stars. (*b*).	Intermediate Stars. (*c*).	Red Stars. (*d*).	Position Angle.	All Types. (*a*).	White Stars. (*b*).	Intermediate Stars. (*c*).	Red Stars. (*d*).
°					°				
10	18	1	2½	1	190	78	20	19	25½
20	22	1½	5½	2	200	75	22	19½	29
30	25	3½	6	6	210	63	17	15	26½
40	28	6½	6½	8	220	58	14	16	17½
50	36	10½	7	8½	230	50	16	13½	12
60	42	8½	7	12	240	39	14	11	9½
70	68	11½	15½	19	250	23	11½	6	6
80	87	12	21½	28	260	17	4	4	6
90	125	25	28	32	270	15	3	2	4
100	144	34½	32	32½	280	15	1½	3	4
110	156	43	38	33½	290	13	2	2	2
120	139	40½	44	29	300	11	1½	3	1½
130	123	30½	42½	32½	310	10	2	2	2
140	110	32½	35½	29	320	7	1	2	1
150	98	24½	30½	29	330	8	½	2	1½
160	77	26	26	17	340	7	0	3	2
170	62	14½	23½	13	350	11	0	2	3
180	70	20	23	14	360	15	1	2½	2

The frequency curves are shown in fig. 2, the scales of the respective diagrams being chosen so as to give each diagram the same total number of stars. The directions of the apices of Drifts I. and II. as derived from the general results in *MN.*, **87**, 122, and the direction of the ant-apex (A.A.) of the solar motion, are given in each of the diagrams.

The main features of the curves are, first, that the white stars, diagram (*b*), and the intermediate stars, (*c*), give very much the same distribution of proper motions as are usually found—as in diagram (*a*)—from an analysis based on the two-stream theory; and, secondly, that the red stars, diagram (*d*), exhibit at first sight and to a very marked extent a separation into two distinctive groupings, the more prominent of which—roughly about the direction of the ant-apex—has a much greater lateral spread than is consistent with the appropriate Drift I. velocity for this part of the sky.

6. In diagram (*d*) I show the direction of Oort's apex ($8^h\ 20^m$, $-44°$) for the high velocity stars. According to Oort * the proportion of high velocity stars of type F is about 1 in 30, for types G and K about 1 in 6, with a much higher proportion for M type dwarfs.

Accordingly, the influence on the high velocity stars in diagram (*b*)

* *Groningen Publications*, No. 40 (Table 14), 1926.

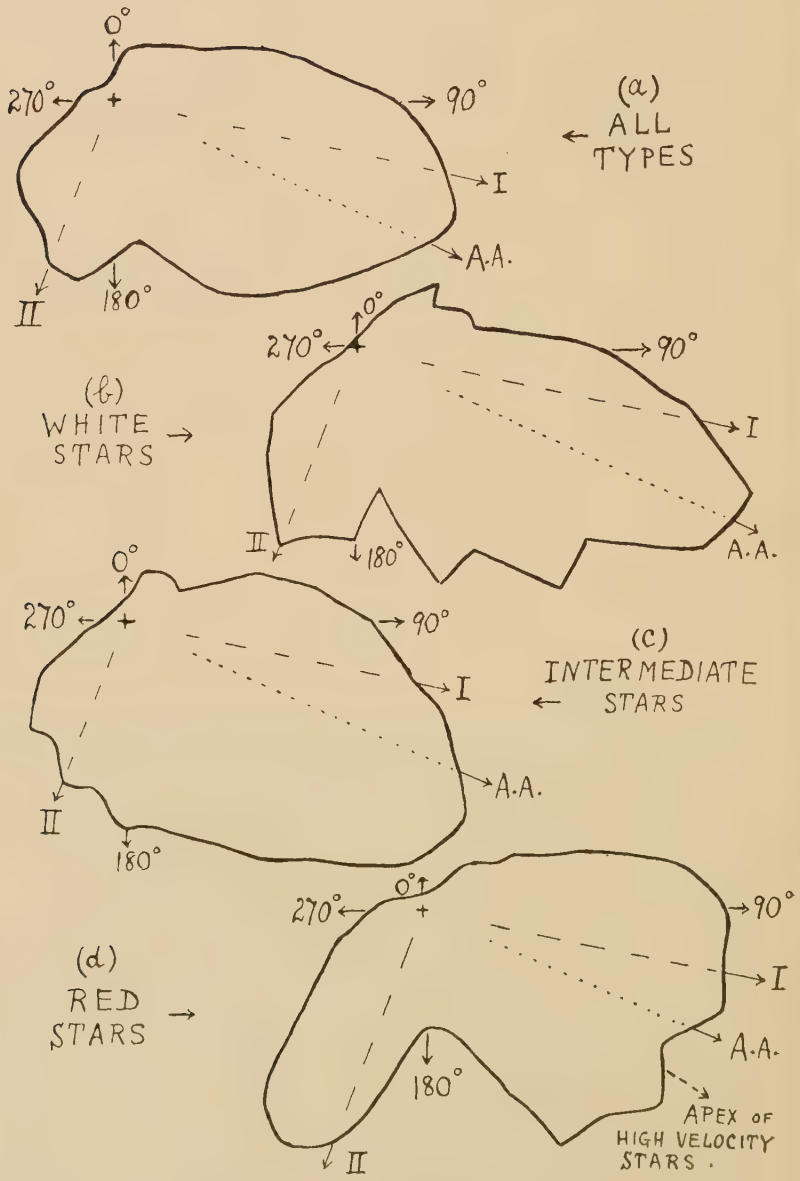

Fig. 2.—Frequency Curves of Proper Motions.

ought to be negligible, fairly weak in (*c*)—there is, however, a slight indication of the effect in this diagram—while in (*d*) the conclusion seems to be that the departure of the diagram from the general characteristics of pure streaming can be explained by the admixture of a considerable proportion of high velocity M dwarfs with the two usual streams. It is to be remembered that the directions of motion of these high velocity stars have a total spread of about 90° on either side of their apex, but as regards their contribution to the statistics of the proper motions shown in diagram (*d*) their effect is hardly likely to be noticeable outside a spread of 30° or so on either side of their apex. For the red stars, diagram (*d*) thus appears to indicate that the star streams are strongly in evidence, together with a not inconsiderable proportion of high velocity stars.

7. The amount of observational material with which we have been dealing is hardly adequate to justify a more elaborate analysis of each of the sub-groups (*b*), (*c*), and (*d*), and the results of this paper are to be regarded as qualitative only. It seems, however, that when we are considering the distribution of proper motions of *all* types—as in diagram (*a*)—the influence of the high velocity stars is small as compared with their effect on the distribution of the proper motions of the red stars, and it is probable that the conclusions which have been reached by various investigators regarding the general systematic motions of faint stars of all types do not require any serious readjustment. Nevertheless it is essential to obtain more precise information concerning the distribution and peculiarities of faint stars of high space velocity, and one of the practicable methods of attack is that indicated in this paper.

In conclusion, I wish to acknowledge the kindness of Dr. Knox-Shaw in taking the filter-photographs and Mrs. Green's assistance in measuring the plates.

Observatory, Cambridge:
1930 *May* 29.

Notes on Galactic Rotation. By J. K. Fotheringham, D.Litt.

In 1926 I contributed to *Monthly Notices*, **86**, 414–26, a determination of the mean rate of rotation of the galactic system, so far as it is represented by the 5413 stars on which Boss based his determination of precession and solar motion.*

I consider that my determination remains the most accurate determination for those stars, and no determination has so far been based on a more complete survey of proper motions in all parts of the sky. My result has since been confirmed by Dr. Oort's discussion of the same stars,† but it has the advantage over all other determinations that it was obtained by introducing an additional unknown into Boss's normal equations, which were so arranged that every quantity was determined independently of the others.

* *Astronomical Journal*, **26** (1910), 95–99, 111–122.
† *B.A.N.*, **4** (1927), No. 132.

Since I wrote, Dr. Oort's discovery of differential rotation * has given a new turn to the subject, and has provided the means of comparing the results obtained from different selections of stars. It appears to me that investigators have made certain assumptions with a precipitancy which is not justified by the accumulated data.

(1) It has been assumed by every investigator except M. Mineur † that the rotation is in the plane of the Milky Way. I find, however, a clearly marked correlation between the mean rate of rotation and the luni-solar precession deduced in the various determinations made from proper motions. Dr. Oort and Sir Frank Dyson use a method in which an accidental error in the difference between the two precessions would produce an error of half that amount in the determined speed of galactic rotation. But, as will be seen, the difference between the two precessions varies very little in the determinations arising from their studies, and I have little doubt that the deduced precession and the deduced galactic rotation contain components of a rotation which is not in the plane of the Milky Way. The values of these components should vary with the mean speed of rotation of the stars selected for discussion, but, if the rotation is in one plane only, the ratio between the components should be constant.

(2) Dr. Oort has been followed by all others in his supposition that the rate of rotation includes the factor cos b where b is the galactic latitude. This is not supported by the evidence. Dr. Oort has determined both the mean rotation and the differential rotation from different galactic zones, opposite zones on different sides of the galactic equator being combined. The coefficients both of the mean rotation and of the differential rotation show a steady progress as we recede from the galactic equator. As will be seen from my table, this progress is no longer conspicuous if we remove this factor by multiplying each of Dr. Oort's results by the mean value of cos b. My own study suggests a progress of a different kind. I find for $u \sin \Omega$, where u is the centennial motion of the pole of the equator in relation to the mean of the stars attributable to the rotation in question and Ω is the right ascension of the ascending node of the plane of rotation, the following values for different zones of galactic latitude :—

Zone.	$u \sin \Omega$.
$+90^\circ$ to $+42^\circ$	$+0''.26$
$+42$,, $+19$	$+0''.62$
$+19$,, $+7$	$+0''.13$
$+7$,, -7	$+0''.18$
-7 ,, -19	$+0''.06$
-19 ,, -42	$-0''.14$
-42 ,, -90	$-0''.13$

* *B.A.N.*, **3** (1927), No. 120.

† *Bulletin astronomique*, 2e série, Mémoires et variétés, Tome v. (1929), 505–43; *M.N.*, **90** (1930), 516–23.

These figures would suggest that the rotation becomes more rapid as you advance towards the north galactic pole, while it becomes less rapid and eventually changes sign as you advance towards the south galactic pole.

It must be remembered, however, that in these determinations from different galactic zones I have not been able to combine opposite parts of the sky with equal weights as in all my other determinations. They are therefore not independent of the values for precession and solar motion given by Boss's equations. I believe, however, that the errors in those values are slight. I have not computed the probable errors of these determinations, and I regard it as not impossible that the apparent progress in the values is accidental.

Dr. Schilt * has found a term of double period in proper motions in right ascension in the three zones $-2°$ to $+1°$, $+24°$ to $+32°$, $+50°$ to $-55°$ Dec., but has been unable to connect it with the galactic latitude. He suggests that a large part of the phenomenon could be accounted for if the systematic motions are different for stars north and south of the galactic plane.

(3) It is supposed that the differential term in the rotation represents the excess of the angular speed of galactic rotation of our sun over the mean angular speed of stars included in the discussion, and that by combining the mean and differential rates of rotation we can discover the sun's speed of rotation and thereby his distance from the galactic centre. If this interpretation were correct we should find that, as we passed from one selection of stars to another, an increase in the one term would be counterbalanced by a decrease in the other term. But as a fact where the one term is large, the other is generally large also, and *vice versa*. It appears also that the size of both terms is affected more by the spectral type of the stars than by their distance or position in the sky. Whether this can be explained by M. Mineur's assumption that there are two rotations which enter in different proportions into different solutions, I am not prepared to say. But I would draw attention to the third harmonic as found by Sir Frank Dyson,† which exceeds the mean rotation for each of the three types of stars investigated by him and exceeds the differential term for two of these three. This proves, at least, the complexity of the subject.

Assuming that there is only one rotation, differing in speed, I have collected the following determinations from proper motions with a view to a determination, so far as is possible, of the extent to which the apparent rotation and apparent precessions are the components of a rotation in a plane other than that of the Milky Way. This ought, among other things, to facilitate the determination of precession. In the case of Sir Frank Dyson's studies I have thought it best to separate the three types B0–B5, B8–A0, A2–A5, and have completed the numerical work for each type. My own determinations of $u \sin \Omega$ are reduced to the plane of the Milky Way by the application of the factor cosec $62°$ cosec $280°$. They are combined for each zone with the precessional

* *Astronomical Journal*, **39** (1929), 143–50.
† *M.N.*, **90** (1929), 235.

terms found by Boss for that zone, and the precessions are then computed as in my former paper. I have omitted my determinations by galactic zones, as Boss's galactic zones do not exactly coincide with mine, and I have therefore no separate determinations of precession for those zones. Dr. Oort's determinations of rotation for the different zones and for the whole sky are reduced by multiplication in each case by the mean value of cos b for the area in question. I have applied the mean value of cos b for the whole sky to Veldt's determination of rotation from faint stars, but I have not applied that factor to values obtained from stars which are selected from the neighbourhood of the galaxy, or which are of types characterised by galactic concentration. Dr. Oort's mean errors have been converted into probable errors. I do not understand how his probable errors for galactic rotation come to be so much smaller than mine, or how on his method they can fall so far short of the probable errors of the precession through which they are determined. No use is made of the probable errors in this paper. In the following table Δp is the deduced correction to Newcomb's centennial precession on the ecliptic, $\Delta\epsilon + \Delta\lambda$ is the combined correction to Newcomb's centennial motion of the equinox and his precession on the equator, B is the mean centennial angular rotation in the plane of the Milky Way, and A is the coefficient of the differential rotation, expressed in seconds of arc per century. $\Delta\epsilon + \Delta\lambda$ is reckoned in the opposite direction to the other terms.

TABLE.

		Δp.	$\Delta\epsilon+\Delta\lambda$.	B.	A.
J. K. F.	All stars	+0″·93±·15	+1″·14±·15	−0″·17±·07	
	0° to ±10° Dec.	+0·58±·40	+0·94±·40	−0·44±·19	
	±10 „ ±30	+0·78±·30	+0·79±·30	−0·21±·14	
	±30 „ ±50	+0·90±·28	+1·38±·28	+0·12±·13	
	±50 „ ±90	+1·21±·28	+1·21±·35	−0·29±·13	
Oort.	All stars	+1·0 ±·1	+1·2 ±·1	−0·18±·04	+0″·14±·05
	0° to ±7° Galactic	+1·7 ±·3	+1·7 ±·3	−0·14±·07	+0·11±·11
	± 8 „ ±19	+1·0 ±·3	+1·1 ±·3	−0·21±·07	+0·19±·11
	±20 „ ±42	+1·0 ±·3	+0·9 ±·3	−0·26±·08	+0·33±·10
	±43 „ ±90	+0·8 ±·3	+1·2 ±·3	−0·15±·08	+0·23±·11
	Bo–B2	+1·4 ±·3	+1·2 ±·3	−0·75±·09	
	B3–B5	+1·3 ±·3	+1·2 ±·3	−0·42±·09	+0·24±·11
	c, O, N, Cepheids	+1·4 ±·3	+1·1 ±·3	−0·42±·09	
Dyson.	Bo–B5	+1·72	+1·72	−0·48	+0·26
	B8–Ao	+1·38	+1·42	−0·29	+0·19
	A2–A5	+2·02	+2·05	−0·33	+1·04
Veldt.*	Faint stars	+1·64±·58	+1·14±·53	−0·69±·36	

* B.A.N., **4** (1928), No. 150.

The large value for A which Sir Frank Dyson has obtained for A2–A5 is confirmed by Dr. Lindblad * from radial velocities. He finds the equivalent of + 0″·70 for the fainter A type stars.

Giving equal weight to each of the seventeen determinations, I have made two analyses. In the one I determine Δp and $\Delta p'/\Delta B$ where $\Delta p'/\Delta B$ is the constant ratio between the excess of the apparent precession on the ecliptic over its true value and the apparent value of rotation in the plane of the Milky Way. In the other analysis I have determined $\Delta\epsilon + \Delta\lambda$ and $(\Delta\epsilon + \Delta\lambda)'/\Delta B$. I obtain

$$\Delta p = + 0''\cdot98 \pm ''\cdot11, \qquad \Delta p'/\Delta B = - 0\cdot78 \pm 0\cdot32.$$
$$\Delta\epsilon + \Delta\lambda = + 1''\cdot28 \pm ''\cdot10, \qquad (\Delta\epsilon + \Delta\lambda)'/\Delta B = + 0\cdot08 \pm 0\cdot26.$$

The two precessions are in close agreement with the results obtained by me in 1926 and by Dr. Oort in 1927. The relation between apparent increments in precession on the ecliptic and apparent rotation in the plane of the Milky Way is well supported, but nothing is proved about the relation of the rotation to the apparent value of precession on the equator, perhaps because the plane of rotation is steeply inclined to the equator, though I suspect that the small value obtained for this term is accidental. The correction to precession on the equator is greatly in excess of what can be attributed to any error in the equinox, and seems to imply, as I suggested in 1926, a motion of the Earth's orbital plane due to some cause other than the known action of the planets.

Assuming 280° for the right ascension of the ascending node of the galactic plane and 62° for its inclination to the equator, I deduce from the ratios found above the following position for the plane of rotation :—

	°
Right ascension of ascending node on equator . .	214·7
Inclination to equator	88·2
Galactic longitude of ascending node on galactic plane	259·5
Inclination to galactic plane	67·4
Ecliptic longitude of ascending node on ecliptic .	36·8
Inclination to ecliptic	72·6

The actual speed of rotation should be 2·6 times as great as its observed component on the galactic plane. The radial velocities give the differential term a coefficient about 1·6 times as great as the coefficient given by the proper motions of the same type of star. This would suggest that the plane of rotation is inclined at an angle of about 51° to the plane of the Milky Way.

In view, however, of the large probable errors attaching to the ratios found in this paper, the deduced position of the plane of rotation may differ from its true position by a large proportion of the total extent of the heavens. It can only serve as an illustration of the direction in which this very tentative study leads.

* M.N., **90** (1930), 508.

University Observatory, Oxford :
1930 *June* 13.

Note.—The discrepancies in the values of $B - A$, which is supposed to give the sun's speed of rotation, would be appreciably reduced if we were to change the sign of A, which we could easily do by shifting the centre of rotation by $\pm$ 90°, placing it near 60° or 240° galactic longitude instead of near 330° as is commonly done. As Dr. Shapley regards 243° as the centre of our local galaxy and 327° as the centre of the super-galaxy, this shift could be associated with a theory that the observed rotation is mainly that of the local galaxy. This is the opinion held by M. Mineur, and Dr. Shapley has endorsed this opinion in his recent paper on the Super-Galaxy Hypothesis.* With an assumed centre near 330° the radial velocities imply that the actual rotation is in a negative direction in galactic longitude. With an assumed centre near 240° they imply that that rotation is in a positive direction. As the apparent direction of the rotation is negative, the former hypothesis implies that the apparent rotation is predominantly direct, the latter that it is predominantly retrograde. We should expect the direct motions to predominate if the sun were near the centre of rotation, and the retrograde motions to predominate if he were at a distance from it. As the prevailing view is that the sun is near the centre of the local galaxy, but remote from the centre of the super-galaxy, the acceptance of either hypothesis seems to involve a revision of existing beliefs. It will be observed that of the nine determinations of A given in my table, six have a smaller absolute value than the corresponding determinations of B, so that in these six cases the two terms are not sufficient to show a retrograde effect in certain directions and a direct effect in other directions. It is probable, however, that part of the rotation effect has been included in the solar motion, and the large size of the third harmonic as found by Sir Frank Dyson suggests the importance of determining more harmonics if we wish to get a fair representation of the total rotation effect.

1930 *June* 27.

Y Cygni. Some Spectroscopic Results.

By R. O. Redman, M.A.

1. *Introduction.*—The eclipsing variable Y Cygni [(1900), $20^h48^m{\cdot}1$ + 34°17′, $7^m{\cdot}1$ – $7^m{\cdot}9$, O9] has been known for some forty-five years. During that time it has been the source of not a little perplexity, but thanks chiefly to the careful researches of Dugan the photometric results at least are now becoming clearer. Full details are not yet available, but it appears definitely established that the period is 2·996332 days, the orbital eccentricity is 0·15, and the line of apsides rotates in a period of about forty-six years.† (These figures are surprisingly close to

* *Harvard College Circular*, 350 (1930), pp. 5, 6.

† Dugan and Waterfield, *H.B.*, 856, 1928.

those found thirty years ago by Dunér, whose work is the more remarkable in view of the fact that he observed by the Argelander method without a photometer.)

The star was observed spectroscopically in 1919–21 by J. S. Plaskett,* who, in view of the position of the photometric work at that time, considered the assumption of a circular orbit justifiable, although he notes that the orbital elements "should not be considered final." Since the results from the light-curve are now more definite and show that the orbit is clearly elliptical, and, further, since Y Cygni was quoted by Russell as the principal example in his recent discussion of the advance of periastron in eclipsing variables,† a revision and extension of the spectroscopic work has become most desirable. This has been undertaken and the results are given here. A more detailed description of the observations, etc., will appear in due course in the Publications of the Dominion Astrophysical Observatory.

2. *Observational Data.*—There were available at Victoria, 35 spectrograms taken in 1919–21 by Plaskett. In addition 39 photographs were taken in 1928–29 by the writer. All spectra were photographed with the Cassegrain spectrograph on the 72-in. reflector at Victoria, B.C., using one prism and either one of two cameras giving dispersions about 29 and 49 A/mm. at H_γ respectively.

The spectrum was classed B2 in the H.D. catalogue and O9·5*n* by Pearce.‡ The lines are double, of approximately equal strength and extremely diffuse and shallow. It is well known that early type eclipsing variables frequently show very nebulous lines; Y Cygni is probably one of the worst in this respect. Interstellar calcium lines are strong and sharp.

On account of the character of the spectrum the early plates were each remeasured twice by the writer, the results being combined with Plaskett's. The 1928–29 plates were measured twice each by the writer only. Between eight and nine stellar lines were used for each component per plate. The interstellar calcium velocities usually depend on the K line only. Ten of the plates were taken at or near times of eclipse and show only single lines. It appears most likely that these are subject to a blend effect§ (and possibly also a rotation effect), which makes them unsuitable for orbital determinations. They have therefore not been used in any of the computations for orbital elements.

The observations are not very well distributed in phase. This is unavoidable, for the double lines can only be separated near the nodes and, owing to the awkward period (five minutes less than three days), even this limited part of the orbit cannot be covered properly unless observations are extended over several seasons.

3. *Orbital Elements.*—The period and phases of the observations were taken from Dugan and Waterfield's figures (*H.B.*, 856, 1926). Four orbits were computed by least squares in the usual way, with results as follows :—

* Pub. *D.A.O.*, **1**, 214, 1920.
‡ Pub. *D.A.O.*, **3**, 298, 1926.
† *M.N.*, **88**, 641, 1928.
§ Shajn and Struve, *M.N.*, **89**, 230, 1929.

Orbit A. 28 plates. 1919–21.

Mean epoch of observations J.D. 2422555·5.
Assumed P = 2·996332 days. Solved for γ, K_1, K_2, e, ω, and T.

$\gamma = -51{\cdot}8 \pm 1{\cdot}4$ (p.e.) km./sec.
$K_1 = 244{\cdot}8 \pm 2{\cdot}4$ km./sec.
$K_2 = 241{\cdot}3 \pm 2{\cdot}4$ km./sec.
$e = {\cdot}136 \pm {\cdot}0098$
$\omega = 4^\circ{\cdot}8 \pm 3^\circ{\cdot}8$
$T = 2{\cdot}407 \pm {\cdot}027$ days.

Mean square residual for plate of unit weight = 12·5 km./sec.

Orbit B. 36 plates. 1928–29.

Mean epoch of observations J.D. 2425644·5.
Assumed P and solved for same elements as in A.

$\gamma = -61{\cdot}8 \pm 1{\cdot}9$ km./sec.
$K_1 = 245{\cdot}0 \pm 3{\cdot}3$ km./sec.
$K_2 = 241{\cdot}5 \pm 3{\cdot}3$ km./sec.
$e = {\cdot}133 \pm {\cdot}011$
$\omega = 86^\circ{\cdot}3 \pm 5^\circ{\cdot}1$
$T = 0{\cdot}005 \pm {\cdot}043$ days.

Mean square residual for plate of unit weight = 20·3 km./sec.

These two orbits are entirely independent of each other and of the photometric work (apart from the assumed period). The agreement between the two sets of values for the K's and e is therefore very good, indeed better than the "probable errors" would lead us to expect. According to the photometric work, $e = 0{\cdot}15$ and the value for ω should have been 6° in orbit A and 72° in orbit B. Bearing in mind the large errors of the radial velocities, consequent on the poor quality of the lines, and remembering the poor distribution of the observations in phase, the spectroscopic figures seem quite satisfactory. The surprising feature of the results is the change in γ. This is discussed later.

Two other orbits were computed, both using all plates. In the first of these the same K's and e are taken throughout, while two sets of values for γ, ω, and T are considered. The solution has then to find only nine unknowns, whereas in the separate orbits A and B the observations are used to obtain twelve altogether.

Orbit C. All plates.

Assumed P as before. Solved for γ', γ'', K_1, K_2, e, ω', ω'', T', T''.
(One dash refers to 1919–21, two to 1928–29.)

$\gamma' = -52{\cdot}0 \pm 2{\cdot}0$ km./sec. $\gamma'' = -61{\cdot}5 \pm 1{\cdot}6$ km./sec.
$K_1 = 244{\cdot}1 \pm 2{\cdot}2$ km./sec.
$K_2 = 241{\cdot}0 \pm 2{\cdot}2$ km./sec.
$e = 0{\cdot}132 \pm 0{\cdot}007$
$\omega' = 6^\circ{\cdot}2 \pm 5^\circ{\cdot}2$ $\omega'' = 83^\circ{\cdot}4 \pm 3^\circ{\cdot}6$
$T' = 2{\cdot}416 \pm 0{\cdot}037$ days $T'' = 2{\cdot}977 \pm 0{\cdot}031$ days.

Mean square residual for plate of unit weight = 17·5 km./sec.

The ω's are in slightly better agreement with the photometric values, but, considering the probable errors, all these figures are virtually the same as those in orbits A and B.

In the final orbit D, the photometric values were taken for ω', ω'' and e.

Orbit D. All plates.

Assumed $\omega' = 6°$, $\omega'' = 72°$ and $e = 0{\cdot}15$, otherwise data and solution exactly as in C.

$\gamma' = -51{\cdot}8 \pm 2{\cdot}0$ km./sec. $\quad \gamma'' = -61{\cdot}7 \pm 1{\cdot}6$ km./sec.

$K_1 = 244{\cdot}9 \pm 2{\cdot}2$ km./sec.
$K_2 = 241{\cdot}5 \pm 2{\cdot}2$ km./sec.

$T' = 2{\cdot}413 \pm 0{\cdot}006$ days $\quad T'' = 2{\cdot}882 \pm 0{\cdot}007$ days.

Mean square residual for plate of unit weight = 17·8 km./sec.

These values are practically the same as those in C, the only appreciable difference being in T″ owing to the change in ω''. The probable errors of T′ and T″ are here considerably lower owing to the fixing of the ω's.

It is to be noticed that the mean square residual is only slightly higher in D than in C. Since the observational material is precisely the same in each case, it is clear that the photometric values for e and the ω's fit the spectroscopic work practically as well as any that can be found.

The residuals from the 1928–29 plates are considerably higher than those from the 1919–21 plates. Three reasons may be given for this : (*a*) the earlier plates were measured by two observers, an important point with a difficult spectrum such as this ; (*b*) the 1928 plates were taken with a smaller dispersion than most of the others ; (*c*) the average weight assigned to a 1928–29 plate was a little higher than the 1919–21 value.

4. *Absolute Dimensions of the System.*—The latest values for the photometric elements are not yet available, but it is unlikely that they will differ very much from those already published. Using the latter, with the spectroscopic results, we find :

	Primary.	Secondary.
Mass	17·4 ⊙	17·6 ⊙
Luminosity . . .	58 per cent. total	42 per cent. total
Semi-major axis of orbit .	$10{\cdot}0 \times 10^6$ km.	$9{\cdot}9 \times 10^6$ km.
Radius	$3{\cdot}3 \times 10^6$ km.	$3{\cdot}3 \times 10^6$ km.
Density	0·16 ⊙	0·16 ⊙

These figures differ little from those given by Plaskett. The chief alteration is in the mass, which is here rather greater. Most of the increase is taken by the secondary which is now slightly more massive than its companion.

The absolute luminosity and parallax may be found by assuming a

surface brightness for the stars. This depends on the effective temperature, of which our knowledge is extremely vague. Above 12,000° or 15,000° there is considerable doubt too concerning the correction from visual to bolometric magnitude. This makes the absolute bolometric magnitude of Y Cygni uncertain, but the parallax may be estimated fairly closely as the following figures show :—

T_e			
15,000°	Bol. Mag. primary	− 2·7	Parallax 0″·0013
20,000°		− 4·0	0″·0010
25,000°		− 4·9	0″·0009

For a mass 17·4⊙ and at these temperatures Eddington's mass-luminosity relation gives bolometric magnitudes − 4·6, − 4·8, − 5·0.

We may take the parallax to be 0″·001 ; the absolute visual magnitude (both components together) is then − 2·9.

The interstellar calcium lines are strong in the spectrum of Y Cygni, indicating a very small parallax. The calcium velocity is well determined at − 9·7±0·8 km./sec. Assuming the calcium to be uniformly distributed in space, its mean distance from us is half that of the star, say 500 parsecs. Then, using data given by Plaskett and Pearce,* the residual calcium velocity left after removing solar motion and galactic rotation is + 1·9 km./sec. This may be regarded as a small peculiar motion of the calcium cloud. Alternatively, if all the observed velocity were attributed to solar motion + "galactic rotation," the mean distance of the calcium would be 700 parsecs, *i.e.* the star's parallax would be 0″·0007, which is probably too small, but is of the same order as the above estimate 0″·001.

5. *The Change in* γ.—We have already noticed that, according to the above orbits, there has been a marked change of about 10 km./sec. in the velocity of the centre of gravity of the system between the years 1919-21 and 1928–29. This may be real or the result of observational error.

Using the probable errors found in orbit C, it may be calculated immediately that the chances are negligible that the observed change in γ is due to a purely accidental error. On the other hand there may have been some systematic difference between the two series of plates. Since all photographs were made with the same instrument, the error, if any, is probably in the measuring. In that case, it is most likely to be found in a systematic difference between Plaskett's and the writer's measures. An examination of the individual results for each plate, in the 1919–21 series, showed a large accidental spread (as the character of the lines leads us to expect) but no very large systematic difference. It seems that the difference $\gamma' - \gamma''$ from this cause is unlikely to exceed 2 km./sec.

We are thus led to the conclusion that much, if not all, of the observed change in γ is real. The most probable explanation is that a third body is present in the system.

Dugan had independently been led to a similar conclusion in a totally

* *M.N.*, **90**, 253, 1929.

different way.* According to Russell's paper, to which reference has already been made, the revolution of periastron can be due to the ellipsoidal form of the two components. But from the light curve Dugan finds the stars to be nearly spherical. This strongly suggests that in the case of Y Cygni the motion of periastron is due to some cause other than the one put forward by Russell. It must be remembered that, if the theory of " radiative viscosity " is correct, the external figure of a star is not necessarily similar to the internal surfaces of equal density, but nevertheless it seems most likely that here an external factor is at work, probably a third star.

6. *The Third Body.*—Assuming that the hypothesis of a third body is correct, what evidence have we concerning its nature, size, position, or velocity? For brevity we will refer to the two principal stars as A and B and the hypothetical third as C. The relevant facts may be summed up as follows :—

(1) A and B rotate round each other in an elliptical orbit, period three days, eccentricity 0·15. They are nearly spherical stars of mass 17·4 ⊙ and 17·6 ⊙.

(2) This elliptical orbit is unvarying except for a revolution of periastron with a period about forty-six years.

(3) The radial velocity of the centre of gravity of A and B has changed by 10 km./sec. in nine years.

(4) No third spectrum has been detected and the light curve of A and B shows that in any case they give most of the light of the system.

The orbit of AB about C must have a fairly high inclination, an orbital velocity of at least 5 km./sec. and a fairly long period, perhaps of the order of twenty years (since otherwise the spectroscopic observations would not have detected the variation of γ). This suggests a fairly large mass for C. But if the latter is stellar, its mass cannot well be very great, because of (4). It seems possible that a star of mass about 5 ⊙ would be sufficiently great to satisfy (3), while its luminosity would be of the order of 1/20 of each of A and B, which would also satisfy (4). This, however, is merely a suggestion, as it appears that more observational data are essential before anything definite can be found concerning C.

7. *Summary.*—A spectroscopic study of the eclipsing variable Y Cygni has been successful in showing an orbital eccentricity and motion of periastron in close agreement with photometric results. The velocity of the centre of gravity of the system has been found to be changing, indicating the presence of a third body. The absolute dimensions and parallax of the two principal stars have been obtained and a brief discussion made of the possible nature and importance of the third star.

I must record my thanks to Dr. J. S. Plaskett who suggested this investigation, and most generously placed at my disposal all the spectrograms and measures made during his earlier work on the star.

* Personal letter.

Victoria, B.C. :
1930 *May.*

On the Dwarf Nature of Spectroscopic Binaries : A Reply.
By E. A. Kreiken.

In a note in the *Bulletin* of the Harvard College Observatory (No. 870, p. 19) Dr. Luyten criticises the results obtained by me in a paper in *M.N.R.A.S.* (**89**, 589, 1929). In that paper I treated all spectroscopic binaries the spectral types of which, both for the main star and its companion, are known. The values $m \sin^3 i$ were plotted against spectral type, and a smooth curve was drawn through the points obtained in this way. The curve thus obtained was brought to coincide with the curve connecting mass and spectral type for the main sequence, by a parallel shift over an interval $\overline{\log \sin^3 i} = \bar{1}{\cdot}905$. The magnitudes of both the components were then derived from Eddington's mass-luminosity relation

$$L = -7/2\Delta \log M - \tfrac{15}{4} \log (1 - \beta) - 2\Delta \log T_e$$

or

$$L = -7/2\Delta \log M \frac{\sin^3 i}{\sin^3 i_0} - \frac{15}{4} \log (1 - \beta) - 2\Delta \log T_e$$

or

$$L = -7/2\Delta \log M - \frac{21}{2}\Delta \,.\, \log M \frac{\sin i}{\sin i_0} - \frac{15}{4} \log (1 - \beta) - 2\Delta \log T_e \quad (1)$$

Since Dr. Luyten objects to my writing $\overline{\log \sin i^3} = \bar{1}{\cdot}905$ or $\overline{\sin i} = 0{\cdot}927$, let us write $\sin i_0 = 0{\cdot}927$. It is then quite evident that the value $\sin i_0 = 0{\cdot}927$ must be used in the expression (1), quite independently of the fact that $\overline{\sin i^3}$ is not equal to $(\overline{\sin i})^3$. Dr. Luyten then proceeds to derive from my data the mean absolute magnitude, and it has evidently escaped his attention that he thus commits the error against which he thinks he is warning me. As stated, the value $\overline{\log \sin i^3} = \bar{1}{\cdot}905$ applied to the observed curve makes it coincide with the theoretical one. For each spectral type the mass corresponding to our ridge line is equal to the dwarf mass of the spectral type in question, and so the magnitude is also equal to the dwarf one. Now let us suppose that for a given star the actual value of the inclination is i; then evidently the true luminosity is

$$L = -7/2\Delta \log M - \frac{21}{2}\Delta \log M \frac{\sin i}{\sin i} - \frac{15}{4} \log (1 - \beta) - 2\Delta \log T_e$$

while it was derived from

$$L_0 = -7/2\Delta \log M - \frac{21}{2}\Delta \log M \frac{\sin i}{\sin i_0} - \frac{15}{4} \log (1 - \beta) - 2\Delta \log T_e.$$

The difference between the true magnitude and the computed one is therefore

$$\Delta m = 2{\cdot}5(\log L - \log L_0).$$

Apparently we cannot expect the mean value of $\overline{\Delta m}$ to be zero. As explained in my paper, positive excesses of i are more frequent than negative ones. If $\sin i$ is smaller than $\sin i_0$, the mass obtained from $\sin i_0$ is also too small and the absolute magnitude too large. Consequently the mean $\overline{\Delta m}$ will be positive. Moreover, the shape of Eddington's curve connecting log mass and absolute magnitude is such, that for the smaller masses the gradient of the curve is larger than for the higher values of the mass. So the amount of error introduced by Luyten is still increased.

The mean magnitude derived in such a way has only a complicated relation to the true mean, which cannot be determined from it unless the distribution of the frequencies of the actual values of $\sin i$ is known. It was for this reason that I did not give this mean in my paper. As Luyten still insists on deriving this mean, I have tried to evaluate the error he makes in this way.

For the spectral type A_0, for instance, we have log -- mass = 0·44 ; mass = 2·8 $\odot$; bol. magn. = + 0·8 ; log $\sin i_0$ = $\bar{1}$·927.

For every star the value of the logarithm of the mass was derived from the expression $\log \frac{m \sin i^3}{\sin {i_0}^3}$. If the actual value is $\sin i$, the resulting error will be $\log \frac{\sin^3 i_0}{\sin^3 i}$.

Suppose the frequency of the values of i to be proportional to $\sin i$; then Table I. gives for intervals ten degrees wide the absolute magnitudes.

TABLE I.

i (Degrees).	3 (log sin i − log sin i_0).	Log mass.	Abs. magn. (bol.).	Sin i.
85	+0·093	+0·53	0·2	0·996
75	+0·054	+0·49	0·5	0·966
65	−0·030	+0·41	1·1	0·906
55	−0·162	+0·28	2·1	0·819
45	−0·354	+0·09	3·8	0·707
35	−0·624	9·82	6·1	0·574
25	−1·023	9·42	9·8	0·423
15	−1·662	8·78	14·1	0·259
5	−3·081	6·36	14·1 (adopted)	0·087

The values in column 4 are taken from Eddington's Table XIV. ("The Internal Constitution of the Stars").

From this we find $\frac{\Sigma m \sin i}{\Sigma \sin i} = + 3{\cdot}24$. The value corresponding to $\sin i_0$ was + 0·8, and so Luyten's error is 2·4 magnitudes.

For the other spectral types the errors are given in Table II.

Table II.

Spectrum.	Error.	Luyten's mean.	Approx. true mean.
Bo–B5	+2·3	0·0	−2·3
B8–B9	+2·4	0·1	−2·3
Ao–A1	+2·4	3·2	+0·8
A2–A3	+2·4	2·4	0·0
A5	+2·5	3·3	+0·8

Luyten will notice that the amount of the error he has made is almost equal to the difference he thought to note between my absolute magnitudes and the usual ones. Luyten's next remark is that spectroscopic binaries are very bright stars and, through this selection, giants. This remark is not to the point. We used stars for which the spectral types of both components are known. The magnitude difference is therefore probably small as is the mass-ratio. So it might even be that we have a selection of absolutely faint stars. This point has been fully discussed in the *M.N.R.A.S.*, **90,** 306, 1930. Finally, Luyten remarks that I included α Aurigæ in my dwarf scheme. It was not included, as is apparent in the different diagrams. It was not specially mentioned, as for this star the value of sin *i* is known. Finally Luyten complains that the star No. 33 of my list is a giant according to my results. Its absolute magnitudes were found to be + 8·1 : and + 8·4 : respectively. I do not think that it is quite usual to consider a star of this absolute magnitude to be a giant as is done by Luyten. Personally I do not think that giant spectroscopic doubles do not occur. I have only tried to prove in various papers that the relative frequency of giants among spectroscopic and visual doubles is less than the frequency of giants among the ordinary stars.

Lembang :
1930 *January.*

The Wave-length of Hε and the Displacements of the Hydrogen Lines in the Sun. By J. Evershed, F.R.S.

Prominence spectra in the H and K region occasionally show a strong image of *Hε*, about 1·6 A. on the less refrangible side of the calcium line H. In measuring over 500 plates for determining the rotation shift and the Einstein effect for the calcium lines, the interval separating H and *Hε* has been measured in all spectra in which the hydrogen line was well enough defined for accurate bisection. The calcium line is always very much stronger than *Hε*, but the relative intensities are far from constant. The lines differ also when there is considerable distortion of H due to motion, for the hydrogen line seems to be less affected. For this reason, only plates showing no appreciable motion are used for these measures.

Only a very small proportion of the total number of plates measured

for H and K gives good images of $H\epsilon$, and up to the present 34 plates have been measured. As the prominences are now diminishing in number and intensity, it seems desirable to record the results obtained without waiting for a larger number of measures.

The mean value of the interval H − $H\epsilon$ from 34 plates is 1·607 A. The average deviation from the mean is 0·005 A., and the probable error ± 0·0008 A. The terrestrial wave-length of H in air and at normal pressure is 3968·471, according to my recent measures.* This is based on the secondary and tertiary standards of the iron arc published in the *Trans. I.A.U.*, **1**, 1922. To reduce this to zero pressure I use the consistent values of the pressure shift obtained by Humphreys at 42, 69, and 101 atmospheres,† which give a mean shift per atmosphere of + 0·0020 A. for H. This line at zero pressure therefore reduces to 3968·469, and hence the wave-length of $H\epsilon$ observed in air, but produced at zero pressure, is 3970·076 ± 0·0008, in close agreement with the value determined by Curtis,‡ which is 3970·075 ± 0·0016 A. But this value is based on Burns's measures of standard iron lines, which differ very slightly in wave-length from the secondary and tertiary standards quoted above, and, assuming that Curtis used six of the Fe lines nearest to $H\epsilon$, a correction of − 0·001 is necessary, which would make his value smaller than mine by 0·002 A. It is to be noted also that my value assumes that the interval H − $H\epsilon$ is the same in the prominences as in terrestrial sources, that is to say, there is no differential shift in the Sun for the two lines. The close agreement indicated shows that this is approximately true, and that $H\epsilon$ is shifted towards the red in the prominences by an amount of the same order as the shift of the calcium line, which I have already determined as + 0·015 A.

At the centre of the Sun, according to the authors of the Revision of Rowland's Preliminary Table, $H\epsilon$ has the wave-length 3970·078, implying a shift in this position of only + 0·002 A. The absorption line is normally very ill-defined and faint, and cannot be measured with accuracy. I have estimated the position from spectra of sunspots when near the centre of the Sun, where the line occasionally appears with increased absorption, or in eruptions as a bright line, and the centre of the absorption or emission appears to me to lie midway between the Cr line 3969·762 and the Fe line 3970·401, which implies the wave-length 3970·081, and a shift of + 0·005 A.

It is now possible from my value of $H\epsilon$ to estimate the wave-lengths and shifts of two of the other less refrangible hydrogen lines by the aid of Balmer's unmodified formula

$$\lambda = \frac{s^2}{a(s^2 - 4)}$$

and the tables of Meggers and Peters of the dispersion of air.§ Computing a, the wave-number of the limit of the Balmer series *in vacuo*,

* *Monthly Notices R.A.S.*, **90**, 189, 1929.
† *Astrophysical Journal*, **26**, 21, 1907.
‡ *Proceedings of the Royal Society*, A, **90**, 613, 1914.
§ *Astrophysical Journal*, **50**, 61, 1919.

I obtain the values of $H\delta$ and $H\gamma$ (reduced to air) given in the table below, in which the shifts of all the hydrogen lines from ϵ to α are compared with the Einstein effect. Curtis's results show that Balmer's formula is only an approximation to the truth, and it fails if extended to $H\beta$ and $H\alpha$. It will, however, give the values of $H\delta$ and $H\gamma$ to practically the same limits of accuracy as that of the measured line.

To obtain the shifts of β and α, I take Curtis's uncorrected values, because for these lines he appears to have used secondary standards which differ only very slightly from the standards published by the *I.A.U.* in 1922.*

Approximate Shifts of the Hydrogen Lines at the Centre of the Sun.

	λ Vacuum.		λ Sun.	Sun − vacuum.	Einstein effect.
$H\varepsilon$	3970·076±·0008		·078	+·002	+·008
$H\delta$	4101·739	computed	·750	+·011	+·009
$H\gamma$	4340·469		·477	+·008	+·009
$H\beta$	4861·326±·0010		·344	+·018	+·010
$H\alpha$	6562·793±·0017		·816	+·023	+·014

The solar wave-lengths are from the Revision of Rowland's Table, which is based on the standards of 1922. The probable errors are not known, and it is doubtful if the values can be trusted to be accurate in the last decimal. The lines are wide and ill-defined, and the difficulties of accurate bisection are increased in the case of $H\delta$ by closely adjacent lines. Perhaps the line $H\gamma$ is the most reliable: it gives a shift nearly in agreement with the Einstein effect. Notwithstanding the errors affecting this table, there can be little doubt that the relative shift increases with wave-length: in the case of $H\epsilon$ it is less than the theoretical, and in the case of $H\alpha$ very much greater. This can readily be explained if we postulate a descending motion of about 0·4 km./sec. in the higher levels of the chromosphere, decreasing to zero at an intermediate level, and changing to a slight ascending motion at the level of $H\epsilon$. For the different lines of hydrogen represent different levels in the chromosphere, $H\alpha$ the highest, $H\gamma$ intermediate, and $H\epsilon$ a low level.

St. John has found that absorption lines originating at a level of 520 km. above the photosphere show displacements in agreement with general relativity. Below this level upward currents exist, and above it his results also indicate a downward movement.†

* It is unfortunate that Curtis does not quote the wave-lengths of the standard lines actually used in his measures, except that in the case of $H\alpha$ he gives in a footnote four wave-lengths of secondary Fe standards used for this line. Of these one is obviously erroneous, owing probably to a misplaced figure: the other three have values on an average 0·003 A. smaller than Burns or the 1922 standards. If a correction is made in this sense, the shift of $H\alpha$ in the Sun is reduced to + 0·020 A.

† *Astrophysical Journal*, **67**, 195, 1928.

Observations of Novæ, 1929–1930. By W. H. Steavenson.

Most of the observations which follow were made with the 6-inch Wray refractor. Where other instruments have been used the estimates have been marked accordingly, as under :

G = 28-inch refractor. Royal Observatory, Greenwich.
G2 = 10-inch refractor. Royal Observatory, Greenwich. (Guider of Astrographic Telescope.)
O = 12·2-inch refractor. University Observatory, Oxford.
R = 12-inch reflector belonging to Dr. Oliver, Ropley, Hants.
E = $4\frac{1}{2}$-inch refractor belonging to Mr. G. E. Ensor, Pretoria.

I am much obliged to the Astronomer Royal, Professor Turner, Dr. Oliver, and Mr. Ensor for permission to use these telescopes.

The observation of Nova Persei on 1930 January 28 (marked K) was made with a 3-inch refractor on the open veldt about 4 miles west of Kimberley. The altitude of the nova was about 12°.

Nova Ophiuchi (1848).

Date.	Mag.
1929 July 12	12·6 (R)

Nova Aquilæ (1918).

1929 Aug. 13	10·8
28	10·8

Nova Cygni (1920).

Date.	Mag.
1929 Sept. 1	15·0
Nov. 3	14·9 (G)

Nova Lacertæ (1910).

1929 Sept. 8	14·3 (G)

Nova Cygni (1876).

Date.	Mag.	Date.	Mag.
1929 Sept. 1	14·71	1929 Sept. 25	15·1 (O)
8	15·1 (G)	Nov. 3	14·71 (G)

Nova Persei (1901).

This nova continues to exhibit marked variability of an irregular type. The extreme range indicated by the observations here recorded is $1^{m}\cdot55$. During 1929 the average brightness was somewhat greater than in recent years ; and on August 13 the nova was more brilliant than I have seen it on any occasion since I began to make regular observations of it in 1915.

[Table.

Date.	Mag.
1929 Aug. 10	11·92
12	12·12
13	11·82
14	12·22
15	12·22
17	12·32
23	12·69
25	12·96
26	12·96
27	13·03
28	12·93
29	12·98
30	12·93
31	12·69
Sept. 1	12·64
3	12·53
4	12·48
5	12·53
6	12·37
7	12·48
8	12·32 (G)
9	12·64
10	12·53
11	12·48
12	12·59
13	12·64
14	12·32
20	13·37
24	13·18
25	13·18 (O)
26	13·03
27	13·03
28	13·13
Oct. 4	12·93
6	13·23
9	13·32
1929 Oct. 12	13·28
14	13·18
15	13·28
16	13·08
23	12·91
25	12·64
26	12·53
27	12·53 (G)
29	12·69
31	12·69
Nov. 3	12·74 (G)
6	12·74
8	12·74
10	12·59 (G2)
12	12·74
13	12·91
20	12·42
21	12·53
23	12·69
25	12·74
26	12·74
30	12·64
Dec. 1	12·64
2	12·59
4	12·74
5	12·74
6	12·64
7	12·64
8	12·53
9	12·74
11	12·64
19	12·74
30	12·32
1930 Jan. 28	12·32 (K)
Feb. 2	12·12 (E)

Three Further Features of the Lunar Crust, from Photographs taken at Dedham. By H. G. Tomkins, C.I.E., F.R.P.S.

Since my paper in December last, weather conditions at Dedham, in common with many other parts of England, have been anything but good, and there have been few nights on which photographs could be taken.

Nevertheless from the plates taken, and others also examined, two further "flow" systems and a very uncommon object found on plates of the moon taken on December 20 last have come to notice. I will deal with the "flow" systems first. Some of their general details are, of course, already known, and they are both of them smaller than the large "flow" dealt with in my last paper.

The first of these extends from the region in the neighbourhood of Birt and the Straight Wall southwards up to and including Pitatus and northwards as far as high ground just east of Lalande. The origin of this "flow" appears to have been in a large partly ruined ring-plain, the remains of which are now occupied by Thebit, Birt, and the Straight Wall. The old rampart of the western half of the ring-plain and the scar round the other half are clear, and the original crater seems to have been a formation about 140 miles in diameter. The formation has been divided by the Straight Wall. This wall, however, is not a ridge, but a fault due to a change of level of the eastern half of the old formation, thus leaving a cliff which at its southern end has been partly broken down. The explanation seems to be that here originally there was a large crater formed by a sub-surface upheaval of the "flow" substance, whatever that may have been, and that due to some cause, probably a line of weakness, half the floor of the crater faulted. This, of course, need not have been a sudden phenomenon, but probably occurred by stages over a considerable period of time, the "flow" substance welling up from below and gradually submerging the eastern half of the crater, together with the walls. It has also flowed over the southern end of the "wall" and invaded the old floor on the western side of the wall. The crater Birt is of later age, as shown by the white ray system emanating from it and extending over the surface of the "flow." It may possibly have been one of the old channels from below subsequently reopened. The above was probably the origin of the "flow," which, it is evident, then proceeded in both south and north directions.

In the former case the stream, having been arrested by the high ground near Pitatus, flooded the region north of that crater and also broke through its northern wall, finally also flooding the crater inside.

The northern branch of the "flow" seems similarly to have flooded the country in that direction, though the portion just round the crater Lassell, being on high ground, escaped, leaving the "flow" to cut its way round and join up again north of that formation. It then seems to have flooded its way on as far as Lalande and there to have been checked by high ground. Some of my plates might possibly be interpreted to indicate a further continuation of the "flow" towards

Copernicus and region west of it, but careful examination seems to be against this, and to indicate rather that one or more other smaller and ndependent floodings may have occurred there, more especially as they are not far from the collection of small flows which are referred to below. It may be added that the double direction of the " flow " is indicated on the plates by marks of erosion along its course in both directions.

I come next to some small flows near to one another and about 100 miles south of Manilius. One of these rises in the crater Boscovich at its south end, crosses the floor of the crater, and then, cutting through its northern wall, goes over the plain outside directly towards Manilius for about 60 miles. The stream is very narrow and distinct, but of small size.

A somewhat similar spot to the one in Boscovich is to be seen on the northern end of the floor of Julius Cæsar, breaking through the northern wall of that crater, and proceeding towards the large " flow " through the Hæmus Mountains, which it eventually joins. I find, however, that this is merely an arm of the main flow from Mare Tranquillitatis, which first cuts through the south-western part of the wall of Julius Cæsar, then passes across the floor and out of that crater, after which it is joined by a second stream proceeding round the outside of the wall of Julius Cæsar in the same direction. This stream therefore belongs to the Nectaris and Tranquillitatis " flow " already reported, and is not an independent feature.

Towards the east, however, is a larger and very peculiarly shaped double flow which from a single source intertwines among two ridges of low irregular hills. The source is a diffuse black spot not marked on the maps, but also shown on one of the Lick Observatory photographs along with the hills. The two branches, after issuing from this source, proceed independently in zigzag courses typical of some kind of fluid stream and at a narrow angle from one another for about 120 miles and then die out.

There is also a third small round spot nearby to the south-east, which seems to have a similar origin but not the stream-like form of its neighbours.

I may, in conclusion, just refer to my third feature, namely, a peculiar appearance near the crater Manzinus caught in the photographic net on the night of December 20 last on the southern terminator. It is cone-shaped and surmounted by a large knob-like object, but its real nature is doubtful and requires further observation at suitable librations, so that I merely mention it here for the present.

Revision of the Measure of the Image of Pluto on the Uccle plate of 1927 January 27.

Professor Stroobant has remeasured the plate of 1927 January 27, on which the image of Pluto was found, and now sends the following correction :—

$$\textit{for} \quad \begin{cases} 7^{h}\ 1^{m}\ 59^{s}\cdot 7 \\ +\ 21^{\circ}\ 17'\ 59''\cdot 5 \end{cases} \qquad \textit{read} \quad \begin{cases} 7^{h}\ 1^{m}\ 59^{s}\cdot 97 \\ +\ 21^{\circ}\ 17'\ 44''\cdot 0. \end{cases}$$